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PROFIT COMPARISON OF TWO STOCHASTIC MODELS EACH PERTAINING TO A TWO-UNIT STANDBY OIL DELIVERING SYSTEM WITH DIFFERENT MODES OF FAILURE

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Abstract: *This paper deals with the profit comparison of two stochastic models A and B each consisting of two unit standby oil delivering system with different modes of failure (i.e., direct failure or via partial failure). In Model 1 the system has only one mode of failure that unit fails completely only by direct failure .In Model II System has two modes of failure i.e. the unit can fails completely either directly from normal mode or via partial failure. Initially one unit is operative and the other is standby. On the complete failure of both the units there is a provision of switching over to the other similar system. This practical situation may be observed in an oil refinery plant. . Techniques of the semi-Markov processes and regenerative processes are used to obtain various measures of system effectiveness and profit incurred. A comparison of model 1 is made with model 2 through graphs.*

Keywords: *Oil delivering system, Semi Markov process, Regenerative point technique, measures of system effectiveness and profit analysis*

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INTRODUCTION

Redundancy technique has been proved as one of the effective strategy to improve performance of repairable systems. Standby systems have been discussed by various researchers including [1-10] under various assumptions/considerations. Some researchers studied some reliability models collecting real data on failure and repair rates of the units used in such systems .The concept of another line facility in case of failure of the operating system have been introduced by Sharma et. al. [8]. This study presents two mathematical models representing oil delivering system having different modes of failure. Model I represents two unit standby oil delivering system with only one mode of failure that unit can fail completely by direct failure The transition diagram of the system is shown in Fig.1. Model II represents the same system configuration as in model I but with one exception that system has two modes of failure that the unit can fail completely either by direct failure from normal mode or via partial failure. The state space diagram of the system is shown in Fig. 2. Other assumptions associated with model I and II are same. Failure time is assumed to have exponential distribution. Repair/Replacement times have been taken as arbitrary certain important results have been derived as compared between two models.

NOTATIONS

O	operative unit
s	stand by
Fr	completely failed unit is under repair
Fwr	completely failed unit is waiting for repair
FR	repair of completely failed unit is continuing from previous state
Frp	completely failed unit is under replacement
Fwrp	completely failed unit is waiting for replacement
FRp	replacement of completely failed unit is continuing from previous state
Frs	repair of completely failed units is kept under suspension
Frps	replacement of completely failed unit is kept under suspension
Pf	unit is under partial failure
Pfr	partially failed unit is under repair
Pfwr	partially failed unit is waiting for repair
PfR	repair of partially failed unit is continuing from previous state
Pfrs	repair of partially failed unit is kept under suspension

C	system gets connection
CV	valve change for being connected
λ	rate of direct complete failure of main pump
λ_1	failure rate of normal to partial failure
λ_2	failure rate of partial to complete failure
α_1	repair rate of completely failed unit
α_2	replacement rate of completely failed unit
α_3	repair rate of partially failed unit
β	rate of change of valve
p	probability that unit is under repair
q	Probability that unit is under replacement
p1	probability of switching over to another line
q1	probability of failure of switching over to another line
G1(t),g1(t)	c.d.f. and p.d.f. of repair time of completely failed unit
G2(t),g2(t)	c.d.f. and p.d.f. of replacement time of completely failed unit
G3(t),g3(t)	c.d.f. and p.d.f. of repair time of partially failed unit

STATE TRANSITION DIAGRAMME

Model1

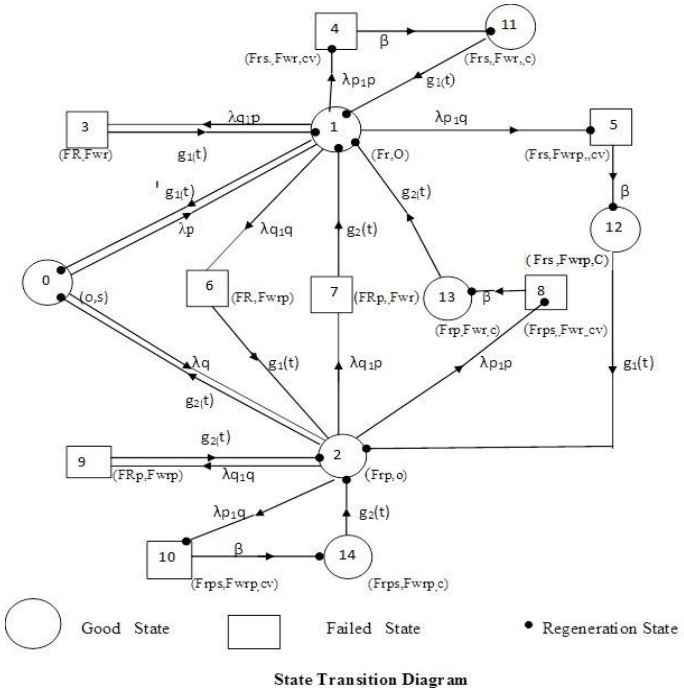

Fig 1 State transition diagram

Model2

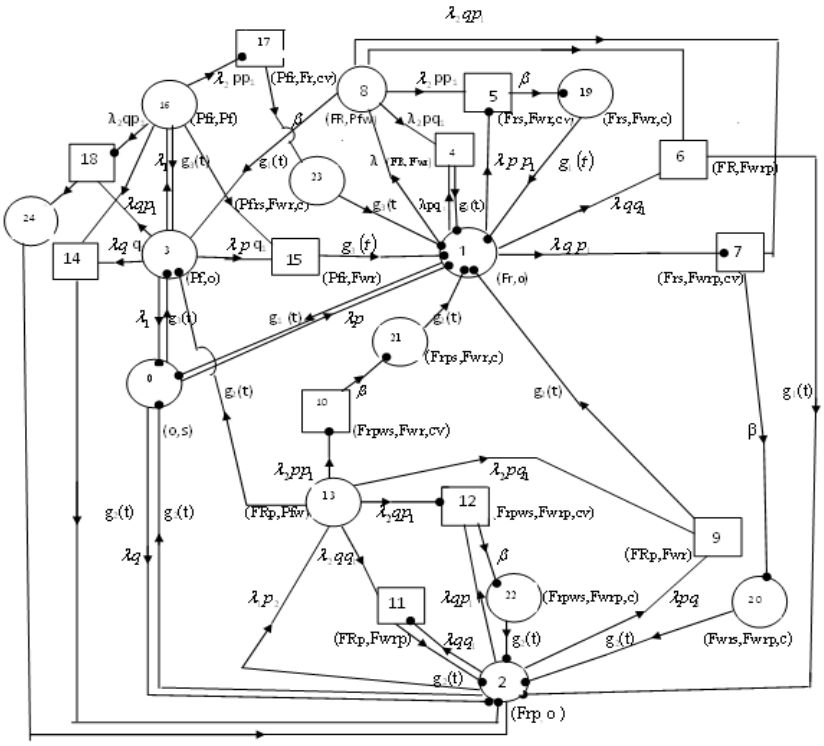

Fig 2 State transition diagram

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Model1

A transition diagram showing the various states of the system is shown in **Fig**

The epochs of entry into states 0, 1, 2, 4, 5, 8, 10,11,12,13 and 14 are regenerative points.

The transition probabilities are given below:

$p_{01} = p$, $p_{02} = q$, $p_{10} = g_1^*(\lambda)$, $p_{11}^{(3)} = q_1 p\{1-g_1^*(\lambda)\}$

$p_{12}^{(6)} = q_1 q\{1-g_1^*(\lambda)\}$, $p_{13} = q_1 p\{1-g_1^*(\lambda)\}$ $p_{14} = pp_1\{1-g_1^*(\lambda)\}$

$p_{15} = p_1 q\{1-g_1^*(\lambda)\}$ $p_{16} = q_1 q\{1-g_1^*(\lambda)\}$ $p_{20} = g_2^*(\lambda)$

$p_{21}^{(7)} = q_1 p\{1-g_2^*(\lambda)\}$ $p_{2,2}^{(9)} = q_1 q\{1-g_2^*(\lambda)\}$ $p_{27} = q_1 p\{1-g_2^*(\lambda)\}$

$p_{28} = p_1 p\{1-g_2^*(\lambda)\}$ $p_{29} = q_1 q\{1-g_2^*(\lambda)\}$ $p_{2,10} = p_1 q\{1-g_2^*(\lambda)\}$

Also μ_i, the mean sojourn tomes in regenerative states i is defined as the time of stay in that state before transition to any other state .Thus

The mean sojourn time (μ_i) in the regenerative state 'i' is given by

$$\mu_0 = \frac{1}{\lambda} \qquad \mu_1 = \frac{1-g_1^*(\lambda)}{\lambda} \qquad \mu_2 = \frac{1-g_2^*(\lambda)}{\lambda} \qquad \mu_4 = \frac{1}{\beta} = \mu_5$$

$\mu_{11} = g_1^{*'}(0) = \mu_{12}$ $\mu_{13} = g_2^{*'}(0) = \mu_{14}$

Model2

A transition diagram showing the various states of the system is shown in **Fig.** The epochs of entry into states 0, 1, 2, 5,7 11,12,17,18,19,20,21 and 22 are regenerative points.

$p_{01} = \frac{p\lambda}{\lambda+\lambda_1}$ $p_{02} = \frac{q\lambda}{\lambda+\lambda_1}$ $p_{03} = \frac{\lambda_1}{\lambda+\lambda_1}$

$p_{10} = g_1^*(\lambda+\lambda_1)$ $p_{14} = \frac{pq_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$ $p_{2,10} = \frac{pp_1\lambda}{\lambda+\lambda_1}\{1-g_2^*(\lambda+\lambda_1)\}$

$p_{11}^{(8,4)} = \frac{q_1p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{11}^{(4)} \frac{pq_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$

$p_{12}^{(8,6)} = \frac{q_1q\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{12}^{(6)} = \frac{qq_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$

$p_{14}^{(8)} = \frac{q_1p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{15} = \frac{pp_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$

$p_{15}^{(8)} = \frac{p_1p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{16} = \frac{qq_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$

$p_{16}^{(8)} = \frac{q_1q\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{17} = \frac{qp_1\lambda}{\lambda+\lambda_1}\{1-g_1^*(\lambda+\lambda_1)\}$

$p_{17}^{(8)} = \frac{p_1q\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_1*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_1*(\lambda_2)}{\lambda_2}\right\}$ $p_{20} = g_2^*(\lambda+\lambda_1)$

$p_{2,9}^{(13)} = \frac{q_1p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_2*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_2*(\lambda_2)}{\lambda_2}\right\}$ $p_{29} = \frac{pq_1\lambda}{\lambda+\lambda_1}\{1-g_2^*(\lambda+\lambda_1)\}$

$p_{2,10}^{(13)} = \frac{p_1p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_2*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_2*(\lambda_2)}{\lambda_2}\right\}$ $p_{2,11} = \frac{qq_1\lambda}{\lambda+\lambda_1}\{1-g_2^*(\lambda+\lambda_1)\}$

$p_{2,11}^{(13)} = \frac{q_1q\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_2*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_2*(\lambda_2)}{\lambda_2}\right\}$ $p_{2,12} = \frac{qp_1\lambda}{\lambda+\lambda_1}\{1-g_2^*(\lambda+\lambda_1)\}$

$p_{2,12}^{(13)} = \frac{qp_1\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_2*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_2*(\lambda_2)}{\lambda_2}\right\}$ $p_{30} = g_3*(\lambda+\lambda_1)$

$p_{3,14}^{(16)} = \frac{q_1q\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_3*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_3*(\lambda_2)}{\lambda_2}\right\}$ $p_{3,14} = \frac{qq_1\lambda}{\lambda+\lambda_1}\{1-g_3^*(\lambda+\lambda_1)\}$

$p_{3,15}^{(16)} = \frac{q_1 p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_3*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_3*(\lambda_2)}{\lambda_2}\right\}$ $p_{3,15} = \frac{pq_1\lambda}{\lambda+\lambda_1}\left\{1-g_3^*(\lambda+\lambda_1)\right\}$

$p_{3,17}^{(16)} = \frac{p_1 p\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_3*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_3*(\lambda_2)}{\lambda_2}\right\}$ $p_{3,17} = \frac{pp_1\lambda}{\lambda+\lambda_1}\left\{1-g_3^*(\lambda+\lambda_1)\right\}$

$p_{3,18}^{(16)} = \frac{qp_1\lambda_1\lambda_2}{\lambda_2-\lambda_1-\lambda}\left\{\frac{1-g_3*(\lambda+\lambda_1)}{\lambda+\lambda_1}-\frac{1-g_3*(\lambda_2)}{\lambda_2}\right\}$ $p_{3,18} = \frac{qp_1\lambda}{\lambda+\lambda_1}\left\{1-g_3^*(\lambda+\lambda_1)\right\}$

MEASURES OF THE SYSTEM EFFECTIVENESS

Various measures of the system effectiveness obtained in steady state using the arguments of the theory of regenerative process are as under:

The Mean Time to System Failure (MTSF) = N/D

The availability of the system (A_0) = N_1/D_1

Busy period analysis of repair time only (B_0) = N_2/D_1

Busy period analysis of replacement time only (BR_0) = N_3/D_1

Expected no of visits by repairman (V_0) = N_4/D_1

Expected no of replacements (R_0) = N_5/D_1

Expected time during which the operation is performed by some other system on the failure of both the units (AP_0) = N_6/D_1

Model1

$N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2$

$D = 1-p_{10}p_{01} - p_{02}p_{20}$

$N_1 = \mu_0((1- p_{11}^{(3)} - p_{14})(1- p_{22}^{(9)} - p_{2,10}) + (p_{12}^{(6)} + p_{15})(- p_{21}^{(7)} - p_{28})) + p_{01}(\mu_0(1- p_{22}^{(9)} - p_{2,10}) + p_{02}(K_2 + \mu_{13}p_{28} + \mu_{14}p_{2,10})) - p_{02}(\mu_0(1- p_{11}^{(3)} - p_{14}) + p_{01}(K_1 + \mu_{11}p_{14} + \mu_{12}p_{13}))$

$D_1 = \mu_0(p_{10} + p_{27} + p_{10}p_{28} + p_{10}p_{20} + p_{20}p_{16} + p_{15}) + K_1(p_{27} + p_{28} + p_{01}p_{20}) + K_2(p_{16} + p_{15} + p_{02}p_{10}) + (\mu_4 + \mu_{11})(p_{27} + p_{28} + p_{20}p_{01})(p_{14} + p_{15}) + (\mu_4 + \mu_{13})(p_{15} + p_{16} + p_{01}p_{02})(p_{28} + p_{2,10})$

$N_2 = (W_1 + W_{11}p_{14} + W_{12}p_{15})(p_{01}P_{20} + p_{27} + p_{28})$

$N_3 = (W_2 + W_{13}p_{28} + W_{14}p_{2,10})(p_{02}p_{10} + p_{15} + p_{16})$

$N_4 = (p_{12}^{(6)} + p_{15})p_{20} + p_{10}(1- p_{22}^{(9)} - p_{2,10})$

$N_5 = 1- p_{11}^{(3)} - p_{14} - p_{01}p_{10}$

$N_6 = (W_{11}p_{14} + W_{12}p_{15})(p_{01}p_{20} + p_{27} + p_{28}) + (W_{13}p_{28} + W_{14}p_{2,10})(p_{02}p_{10} + p_{12}^{(6)} + p_{15})$

Model2

$$\boldsymbol{N} = (1-p_{3,3}^{(16)})(\mu_1 + p_{01}K_1 + p_{02}K_2) + K_3\,(p_{02}\,p_{2,3}^{(13)} + p_{03} + p_{0,1}\,p_{13}^{(8)})$$

$$\boldsymbol{D} = 1 - p_{02}\,p_{2,3}^{(13)}p_{3,0} - p_{03}\,p_{30} - p_{01}\,p_{1,3}^{(8)}p_{30} - p_{10}\,p_{01} - p_{20}\,p_{02} + p_{3,3}^{(16)}(p_{10}\,p_{01} - 1 + p_{20}\,p_{02})$$

$$\boldsymbol{N_1} = \mu_0\,((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))(1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))$$
$$(1-p_{3,3}^{(16)}) + p_{2,3}^{(13)}(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))(-$$
$$p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(1-p_{3,3}^{(16)}) + p_{2,3}^{(13)}(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})) -$$
$$p_{1,3}^{(8)}(-p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(-\ p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (1-p_{2,2}^{(11)} -$$
$$p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))(-\ p_{3,1}^{(15} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))) + p_{0,1}((\mu_1 + \mu_{20}(p_{17}^{(8)} + p_{17})$$
$$+\mu_{19}(p_{15}^{(8)} + p_{15}))((1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))(1-p_{3,3}^{(16)}) + p_{2,3}^{(13)}(-p_{3,2}^{(14)} -$$
$$p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))((\mu_2 + \mu_{21}(p_{2,10}^{(13)} + p_{2,10}) + \mu_{22}(p_{2,12}^{(13)})$$
$$+ p_{2,12}))(1-p_{3,3}^{(16)}) + p_{2,3}^{(13)}(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18}))) - p_{1,3}^{(8)}((\mu_2 + \mu_{21}(p_{2,10}^{(13)})$$
$$+ p_{2,10}) + \mu_{22}(p_{2,12}^{(13)} + p_{2,12}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} -$$
$$(p_{2,12}^{(13)} + p_{2,12}))(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18}))) - p_{0,2}((\mu_1 + \mu_{20}(p_{17}^{(8)} + p_{17}) +$$
$$\mu_{19}(p_{15}^{(8)} + p_{15}))((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(1-p_{3,3}^{(16)}) + p_{2,3}^{(13)}(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} -$$
$$(p_{3,17}^{(16)} + p_{3,17})) - (1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))((\mu_2 + \mu_{21}(p_{2,10}^{(13)} + p_{2,10}) + \mu_{22}(p_{2,12}^{(13)} + p_{2,12}))(1-$$
$$p_{3,3}^{(16)}) + p_{2,3}^{(13)}(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18}))) - p_{1,3}^{(8)}((\mu_2 + \mu_{21}(p_{2,10}^{(13)} + p_{2,10}) +$$
$$\mu_{22}(p_{2,12}^{(13)} + p_{2,12}))(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})) - (p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))$$
$$(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18})))) + p_{0,3}((\mu_1 + \mu_{20}(p_{17}^{(8)} + p_{17}) + \mu_{19}(p_{15}^{(8)} + p_{15}))((p_{2,1}^{(9)} -$$
$$p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} +$$
$$p_{2,12}))(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))) - (1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))((\mu_2 +$$
$$\mu_{21}(p_{2,10}^{(13)} + p_{2,10}) + \mu_{22}(p_{2,12}^{(13)} + p_{2,12}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (1-p_{2,2}^{(11)} -$$
$$p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18}))) + (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)}$$
$$+ p_{17}))((\mu_2 + \mu_{21}(p_{2,10}^{(13)} + p_{2,10}) + \mu_{22}(p_{2,12}^{(13)} + p_{2,12}))(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})) -$$
$$p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(\mu_3 + \mu_{23}(p_{3,17}^{(18)} + p_{3,17}) + \mu_{24}(p_{3,18}^{(16)} + p_{3,18})))$$

$$\boldsymbol{D_1} = \mu_0(p_{10}((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} +$$
$$p_{3,18})) - (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))) - p_{20}((1-$$
$$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))$$
$$(-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))) + p_{30}((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))(1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)}$$
$$- (p_{2,12}^{(13)} + p_{2,12})) - (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))(p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))) + K_4(-$$
$$p_{01}p_{20}(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) + p_{01}p_{30}(1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})) +$$

$p_{02}p_{20}$ ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$) - $p_{02}p_{30}$ ($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$)+ ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)($-p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$)- ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)(1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$) + K_5(p_{01} (p_{10} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$) - p_{30}($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$))-p_{02}(($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$) p_{10} + p_{30}(1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$))- (1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$) ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)+($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$)) + K_6(p_{01}(p_{10} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)-p_{20} ($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$)) + p_{02}(p_{10} (-$p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$) - p_{20}(1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$))+ (1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$) (1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$)-($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$) ($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$)) + μ_5($p_{15}^{(8)}$+p_{15})(((1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$) - $p_{02}p_{20}$)(1-$p_{3,3}^{(16)}$) + p_{03}(p_{20} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$) - p_{30} (1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$))+ $p_{23}^{(13)}$(($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$) - $p_{02}p_{30}$)) + μ_7(p_{17} + $p_{17}^{(8)}$) (p_{03}(p_{30} ($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$) - p_{20} ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)) + $p_{23}^{(13)}$(p_{01} p_{30} - ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)) +(1-$p_{33}^{(16)}$)($p_{01}p_{20}$-($p_{2,1}^{(9)}$–$p_{2,1}^{(13,9)}$-($p_{2,10}^{(13)}$ + $p_{2,10}$))) + μ_{10}($p_{2,10}$+$p_{210}^{(13)}$)($p_{1,3}^{(8)}$(p_{02} p_{30}-($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$))+(1-$p_{3,3}^{(16)}$)(p_{02} p_{10} - ($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$))+ p_{03}(p_{30} ($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$) - p_{10} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)) + μ_{12}($p_{2,12}$+$p_{2,12}^{(13)}$)(p_{03}(p_{10} ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)-p_{30}(1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$))+ $p_{1,3}^{(8)}$(($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)-p_{01} p_{30})+(1-$p_{3,3}^{(16)}$)((1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$)-$p_{01}p_{10}$))+ μ_{17} ($p_{3,17}$+$p_{3,17}^{(16)}$)($p_{1,3}^{(8)}$)((1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$)-$p_{02}p_{20}$)+ p_{03}(p_{10} (1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$)-p_{02} ($-p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$))+$p_{2,3}^{(13)}$(p_{02}-(-$p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$ p_{10})) + μ_{18}($p_{3,18}$+$p_{3,18}^{(16)}$($p_{1,3}^{(8)}$($p_{01}p_{20}$-($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$))+p_{03}(p_{20}(1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$) - p_{10} ($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$))+$p_{2,3}^{(13)}$((1-$p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})$)-$p_{01}p_{10}$))+ μ_{19} (p_{15}+$p_{1,5}^{(8)}$) (p_{03}(p_{20} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)-p_{30} (1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})$))+ $p_{2,3}^{(13)}$(($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$)-$p_{02}p_{30}$)+(1-$p_{3,3}^{(16)}$)((1-$p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - p_{12,22}$ $p_{22,2}$ ($p_{2,12}^{(13)}$ + $p_{2,12}$))-$p_{02}p_{20}$))+μ_{20}(p_{17}+$p_{1,7}^{(8)}$(p_{03}(p_{30} ($p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})$)-p_{20} ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$))+ $p_{2,3}^{(13)}$($p_{01}p_{30}$-($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$))+(1-$p_{3,3}^{(16)}$)($p_{01}p_{20}$-($p_{2,1}^{(9)}$–$p_{2,1}^{(13,9)}$ -($p_{2,10}^{(13)}$ + $p_{2,10}$)))+μ_{21}($p_{2,10}$+$p_{2,10}^{(13)}$($p_{1,3}^{(8)}$($p_{0,2}p_{3,0}$-($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$))+(1-$p_{3,3}^{(16)}$)($p_{02}p_{10}$-($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$))+p_{03}(p_{30} ($p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})$)-p_{10} ($-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})$))+μ_{22}($p_{2,12}$+$p_{2,12}^{(13)}$(p_{03}(p_{10} ($-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})$)-p_{30}(1-$p_{11}^{(4)}$ -

$p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})))+p_{1,3}^{(8)}((-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))-p_{0,1}p_{3,0})+(1-p_{3,3}^{(16)})((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))-p_{0,1}p_{1,0}))+\mu_{23}(p_{3,17}+p_{3,17}^{(16)}(p_{1,3}^{(8)}((1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))-p_{0,2}p_{2,0})+ p_{0,3}(p_{1,0} (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))-p_{0,2} (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})))+ p_{2,3}^{(13)}(p_{0,2}p_{1,0}-(p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))))+\mu_{24}(p_{3,18}+p_{3,18}^{(16)})(p_{03}(p_{2,0}(1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))-p_{1,0} (p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})))+p_{2,3}^{(13)}((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))-p_{0,1}p_{1,0})+p_{1,3}^{(8)}(p_{0,1}p_{2,0}-(p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))))$

$\boldsymbol{N_2} = (W_1+\mu_{19}(p_{15}+p_{15}^{(8)})+\mu_{20}(p_{17}+p_{17}^{(8)}))(-p_{01}((1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})) (1-p_{3,3}^{(16)}+ p_{2,3}^{(13)} (- p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})))+p_{02}((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))(1-p_{3,3}^{(16)} + p_{2,3}^{(13)} (- p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})))-p_{03}((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})) (- p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})) - (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})) (-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17}))))) (W_3+\mu_{23}(p_{3,17}+p_{3,17}^{(16)})+ \mu_{24}(p_{3,18}+p_{3,18}^{(16)})) (- p_{01}(-p_{23}^{(13)} (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})) + p_{13}^{(8)} (1- p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))) +p_{02}(- p_{23}^{(13)} (1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))+p_{13}^{(8)} (p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})))-p_{03}((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})) (1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))-(p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})) (p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10}))))$

$\boldsymbol{N_3} = (W_2+(p_{2,10}+p_{2,10}^{(13)}) \mu_{21}+\mu_{22}(p_{2,12}+p_{2,12}^{(13)}))(-p_{01}((p_{12}^{(6)} - p_{12}^{(8,6)}) - (p_{17}^{(8)} + p_{17})) (1-p_{3,3}^{(16)})+p_{1,3}^{(8)} (- p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18})))+ p_{02}((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})) (1-p_{3,3}^{(16)})+p_{13}^{(8)} (-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})))-p_{03}((1-p_{11}^{(4)} - p_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15}))(-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18}))-(p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17})) (- p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})))))$

$\boldsymbol{N_4} = ((1-p_{11}^{(4)} -p_{11}^{(8,4)} - (p_{15}^{(8)} +p_{15}))((1-p_{2,2}^{(11)} -p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12}))(1-p_{3,3}^{(16)}) +p_{2,3}^{(13)} (-p_{3,2}^{(14)} -p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} +p_{3,18})))-(p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} +p_{17}))((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} +p_{2,10}))(1-p_{3,3}^{(16)}+p_{2,3}^{(13)} (-p_{3,1}^{(15)} -p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} +p_{3,17})))-p_{1,3}^{(8)}((p_{2,1}^{(9)} - p_{2,1}^{(13,9)} - (p_{2,10}^{(13)} + p_{2,10})) (-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} +p_{3,18}))-(1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} - (p_{2,12}^{(13)} + p_{2,12})) (-p_{3,1}^{(15)} -p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})))))$

$\boldsymbol{N_5} = (-p_{01}((p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} + p_{17}))(1-p_{3,3}^{(16)}) + p_{1,3}^{(8)} (-p_{3,2}^{(14)} -p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} +p_{3,18}))) + p_{02}((1-p_{1,1}^{(4)} - p_{1,1}^{(8,4)} - (p_{15}^{(8)} +p_{15}))(1-p_{3,3}^{(16)}) + p_{1,3}^{(8)} (-p_{3,1}^{(15)} -p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} +p_{3,17})))-p_{03}((1-p_{11}^{(4)} - P_{11}^{(8,4)} - (p_{15}^{(8)} + p_{15})) (-p_{3,2}^{(14)} -p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} +p_{3,18})) - (p_{12}^{(6)} - p_{12}^{(8,6)} - (p_{17}^{(8)} +p_{17})) (-p_{3,1}^{(15)} - p_{3,1}^{(15,16)} - (p_{3,17}^{(16)} + p_{3,17})))))$

$\boldsymbol{N_6} = p_{01}((\mu_{20}(p_{17}+p_{17}^{(8)})+ \mu_{19}(p_{15}+p_{15}^{(8)}))((1-p_{2,2}^{(11)} - p_{2,2}^{(13,11)} -(p_{2,12}^{(13)} + p_{2,12}))(1-p_{33}^{(16)})+p_{23}^{(13)} (-p_{3,2}^{(14)} - p_{3,2}^{(16,14)} - (p_{3,18}^{(16)} + p_{3,18}))-(p_{12}^{(6)} - p_{12}^{(8,6)} -(p_{17}^{(8)}+p_{17}))($

$\mu_{22}(p_{2,12}^{(13)}+p_{2,12})+\mu_{21}(p_{2,10}+p_{2,10}^{(13)})(1-p_{3,3}^{(16)}+p_{2,3}^{(13)}(\mu_{23}(p_{3,17}+p_{3,17}^{(16)})+\mu_{24}\ p_{3,18}+p_{3,18}^{(16)})))-p_{13}^{(8)}(\ \mu_{22}(p_{2,12}^{(13)}+p_{2,12})+\ \mu_{21}(p_{2,10}+p_{2,10}^{(13)})(\ -p_{3,2}^{(14)}\ -\ p_{3,2}^{(16,14)}\ -(p_{3,18}^{(16)}\ +\ p_{3,18}\ ))-(\ p_{2,1}^{(9)}\ -\ p_{2,1}^{(13,9)}\ -\ \ (p_{2,10}^{(13)}+p_{2,10}))(\ \mu_{23}(p_{3,17}+p_{3,17}^{(16)})\ +\ \mu_{24}(p_{3,18}+p_{3,18}^{(16)})))))-p_{02}((\ \mu_{20}(p_{17}+p_{17}^{(8)})+\mu_{19}(p_{15}+p_{15}^{(8)}))((\ p_{2,1}^{(9)}(s)-p_{2,1}^{(13,9)}-(p_{2,10}^{(13)}+p_{2,10}\ )\ )(1-p_{3,3}^{(16)*})+p_{2,3}^{(13)*}\ (\ -p_{3,1}^{(15)}\ -\ p_{3,1}^{(15,16)}\ -(p_{3,17}^{(16)}\ +\ p_{3,17}\ )))-(\ 1-p_{11}^{(4)}\ -\ p_{11}^{(8,4)}\ -(p_{15}^{(8)}\ +\ p_{15}\ )\ )(\ \mu_{22}(p_{2,12}^{(13)}+p_{2,12})+\mu_{21}(p_{2,10}+p_{2,10}^{(13)})(1-p_{3,3}^{(16)})\ +\ p_{2,3}^{(13)}(\mu_{23}(p_{3,17}+p_{3,17}^{(16)})+\mu_{24}(p_{3,18}+p_{3,18}^{(16)})))-p_{13}^{(8)}(\ \ (\mu_{22}(p_{2,12}^{(13)}+p_{2,12})+\ \mu_{21}(p_{2,10}+p_{2,10}^{(13)}))(\ -p_{3,1}^{(15)}\ -\ p_{3,1}^{(15,16)}\ -\ (p_{3,17}^{(16)}\ +\ p_{3,17}\ ))-(\ p_{2,1}^{(9)}\ -\ p_{2,1}^{(13,9)}\ -(p_{2,10}^{(13)}+p_{2,10}))\ (\ \mu_{23}(p_{3,17}+p_{3,17}^{(16)})\ +\ \mu_{24}(p_{3,18}+p_{3,18}^{(16)}))))\ +\ p_{03}((\ \mu_{20}\ (p_{17}\ +p_{17}^{(8)})+\ \mu_{19}(p_{15}+p_{15}^{(8)}))((\ p_{2,1}^{(9)}\ -\ p_{2,1}^{(13,9)}\ -\ (p_{2,10}^{(13)}+p_{2,10}\ )\ )\ (\ -p_{3,2}^{(14)}\ -p_{3,2}^{(16,14)}\ -(p_{3,18}^{(16)}+p_{3,18}\ ))-(1-p_{2,2}^{(11)}\ -\ p_{2,2}^{(13,11)}\ -(p_{2,12}^{(13)}\ +\ p_{2,12}\ )\ )(\ -p_{3,1}^{(15)}\ -\ p_{3,1}^{(15,16)}\ -\ (p_{3,17}^{(16)}+\ p_{3,17}\ )))-(\ 1-p_{11}^{(4)}\ -\ p_{11}^{(8,4)}\ -(p_{15}^{(8)}+p_{15}))(\ \mu_{22}(p_{2,12}^{(13)}+\ p_{2,12})+\ \mu_{21}(p_{2,10}+p_{2,10}^{(13)})\ (\ -p_{3,2}^{(14)}\ -\ p_{3,2}^{(16,14)}\ -\ (p_{3,18}^{(16)}\ +\ p_{3,18}\ ))-(1-p_{2,2}^{(11)}\ -\ p_{2,2}^{(13,11)}-(p_{2,12}^{(13)}+p_{2,12}))(\ \mu_{23}\ (p_{3,17}+p_{3,17}^{(16)})+\ \mu_{24}(p_{3,18}+p_{3,18}^{(16)})))+(\ p_{12}^{(6)}\ -\ p_{12}^{(8,6)}\ -(p_{17}^{(8)}\ +\ p_{17}\ )\ )(\ \mu_{22}(p_{2,12}^{(13)}+p_{2,12})+\ \mu_{21}(p_{2,10}+p_{2,10}^{(13)*})(\ -p_{3,1}^{(15)}\ -\ p_{3,1}^{(15,16)}\ -\ (p_{3,17}^{(16)}+\ p_{3,17}\ ))-(\ p_{2,1}^{(9)}\ -\ p_{2,1}^{(13,9)}\ -\ (p_{2,10}^{(13)}+\ p_{2,10}\ )\ )\ (\mu_{23}(p_{3,17}+p_{3,17}^{(16)})+\ \mu_{24}(p_{3,18}+p_{3,18}^{(16)}))))$

PROFIT ANALYSIS

Expected Profit = $C_0A_0 - C_1B_0 - C_2BR_0 - C_3V_0 - C_4R_0 - C_5AP_0$

C_0 = revenue per unit up time

C_1 = cost per unit time for which the repairman is busy for repair

C_2 = cost per unit time for which the repairman is busy for replacement

C_3 = cost per visit of the repairman

C_4 = cost per unit replacement

C_5 = cost per unit of alternate performance

PARTICULAR CASE FOR MODEL 1 AND MODEL 2

The following particular case is considered for graphical interpretation

$g_1(t) = \alpha_1 e^{-\alpha_1 t}$ $g_2(t) = \alpha_2 e^{-\alpha_2 t}$ $g_3(t) = \alpha_3 e^{-\alpha_3 t}$

GRAPHICAL REPRESENTATION OF RESULTS

Model1

The behavior of profit with respect to revenue C_0 for different values of cost of alternate performance C_5 is shown as in Fig. 1.3. It is clear from the graph that profit increases with the increase in the values of C_0 but decrease as the value of alternate performance C_5

increases. Fig 1.4 shows the behavior of Profit with respect to cost per visit of repairman C_4 for different values of cost for busy period of repairman C_1.It is clear from the graph that profit decreases with the increase in cost per visit of repairman C_4 and also with increase in cost of busy period of repairman C_1.

Model2

The behavior of profit with respect to revenue C_0 for different values of cost of busy period of repairman C_1 is shown in fig 1.5. It is clear from the graph that profit increases with the increase in the values of C_0 and decreases as the value cost of busy period of repairman C_1 increases. Fig 1.6 shows the behavior of profit with respect to cost per visit of repairman C_4 for different values of cost of busy period of repairman C_1. It is clear from the graph that profit decreases with the increase in cost per visit of repairman C_4 and also with increase in cost of busy period of repairman C_1.

COMPARISON BETWEEN THE MODELS

Comparison between the models shows the behavior of the difference between the profits, i.e., $P_1 - P_2$ with respect to revenue per unit up time (C_0) of an oil delivering system for different values of cost per unit time for which the repairman is busy for repair **(C_1)**

From the graph, following is concluded:

(i) The difference $P_1 - P_2$ increases as the revenue (C_0) increases. Also, the difference becomes higher for higher values of cost (C_1).

(ii) For C_1 =10, $P_1 - P_2$ > or = or < 0 according as C_0 > or = or < 34.116. Hence Model 1is better or worse than the model 2 whenever C_0 > or < 34.116. Both the models are equally good if C_0 = 34.116.

(iii) For C_1 = 50, $P_1 - P_2$ > or = or < 0 according as C_0 > or = or < 31.69. Hence, Model 1 is better or worse than the Model 2 whenever C_0 > or < 31.69. Both the models are equally good if C_0 = 31.69.

(iv) For C_1 = 100, $P_1 - P_2$ > or = or < 0 according as C0 <or = or > 28.66. Hence Model 1 is better or worse than the Model 2 whenever C_0 > or < 28.66. Both the models are equally good if C_0 = 28.66.

Fig.1.7 shows the behavior of the difference between the profits, i.e., $P_1 - P_2$ with respect to cost per unit of repairman (C_4) of an oil delivering system for different values of cost of alternate performance (C_5)

From the graph, following is concluded:

(i) The difference $P_1 - P_2$ increases as the cost (C_4) increases. Also, the difference becomes lower for higher values of cost (C_5).

(ii) For C_5 =500, $P_1 - P_2$ > or = or < 0 according as C4 > or = or < 20211.835. Hence Model 1is better or worse than the Model 2 whenever C_4 > or < 20211.835. Both the models are equally good if C_4 = 20211.835.

(iii) For C_5 = 520, $P_1 - P_2$ > or = or < 0 according as C_4 > or = or < 21435.18.. Hence, Model 1 is better or worse than the Model 2 whenever C_0 > or < 21435.18. Both the models are equally good if C_4 = 21435.18.

(iv) For C_5 = 540, $P_1 - P_2$ > or = or < 0 according as C4 <or = or > 22658.525. Hence Model 1 is better or worse than the Model 2 whenever C_0 > or < 22658.525. Both the models are equally good if C_4 = 22658.525.

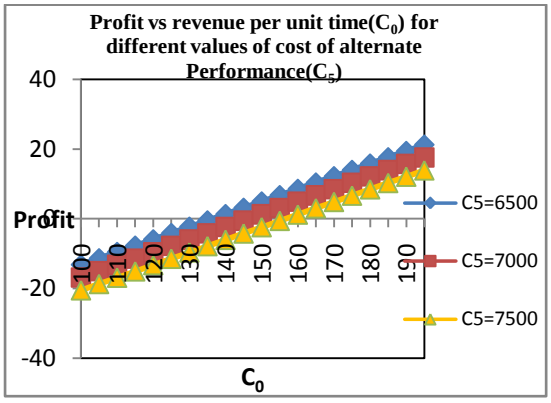

Fig. 1.3

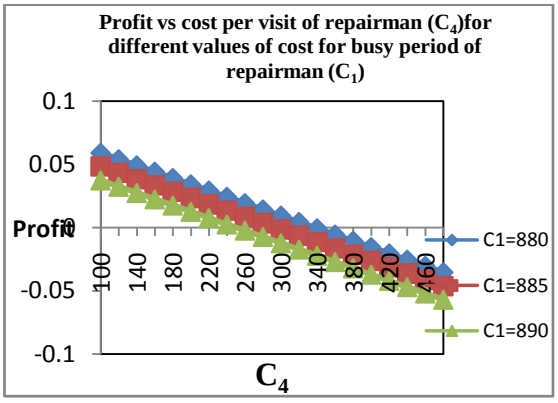

Fig.1.4

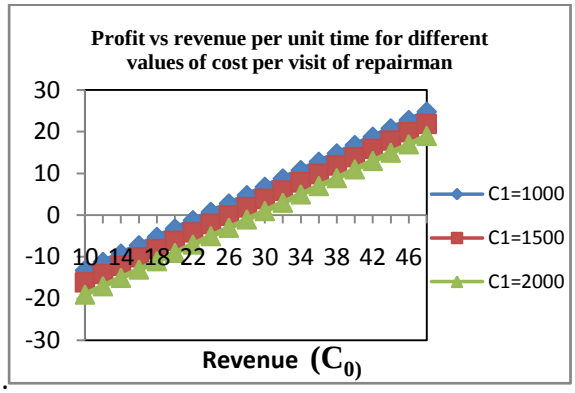

Fig1.5

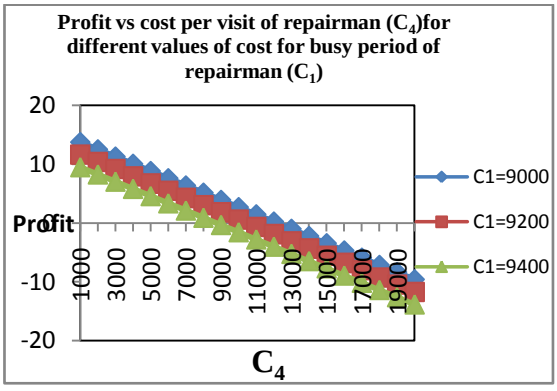

Fig.1.6

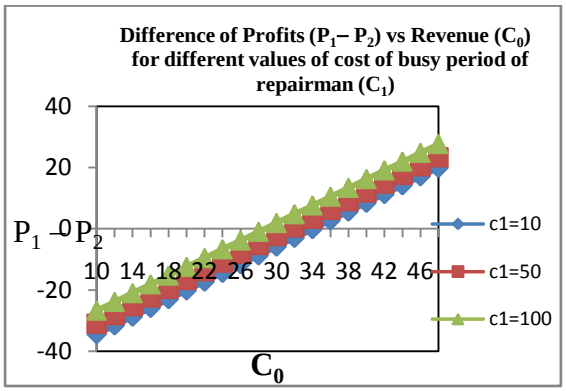

Fig 1.7

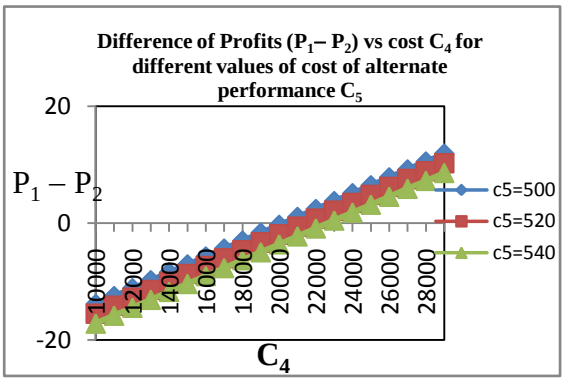

Fig. 1.8

REFERENCES

[1] Goel , l.R.,Gupta R. Moafim B.S.E .," Operating orbit system with two-dissimilar units and corresponding standby," *Int. J. Systems Sci.,* vol. 21, pp. 495-500, 1990.

[2] Barlow, R.E. and Proschan, F., "Mathematical Theory of Reliability," *John Wiley, New York*. 1965.

[3] Goyal, V., "Fundamental models with various types of repair facilities *in the theory of reliability*," Ph.D. Thesis, University of Roorkee, Roorkee, 1984.

[4] Rander, M.C., Kumar, A. and Tuteja, R.K.," A two-unit cold standbysystem with major and minor failures and preparation time in the case of major failure." *Microelectron. Reliab., 32,*pp 1199-1204. 1991.

[5] Taneja, G.," Stochastic and profit analysis of some reliability models with different types of failure and repair." *Ph.D. Thesis, M.D.U.,Rohtak*, 1992.

[6] Tuteja, R.K. and Taneja, G., "Profit analysis of a one-server one-unit system with partial failure subject to random inspection."*Microelectron. Reliab*., 33,;pp 319-322., 1993.

[7] Yamashem, M., "A repairable multiple state device with general repair time", *IEEE, Trans. Reliab., R-29*, 1980; 276.

[8] Amit Goyal, Taneja G., Singh D.V, ."Reliability and profit evaluation of a 2-unitcold standby system working in a sugar mill with operating and rest periods,"*in Caledonian Journal of Engineering,vol 1-5,. 2009*

[9] U.Sharma,Rekha, G Taneja., " Analysis of aTwo Unit Standby Oil Delivering System with a Provision of Switching Over to another System at Need to Increase the Availability", in *Journal of Mathematics and Statistics*" Vol 7 NO 1, 2011, pp 57-60.

[10] Archana Sharma, Jai Singh, Kuldeep Kumar "Availability Evaluation of The Serial Processes in a Paper Production Industry-A Numerical Approach," *in Mathematical Sciences Quarterly Journal,* VOL 4(4), 2010 ,pp 445-465.

[11] Rizwan S.M. "Reliability analysis of a twounit system with two repairmen", *in Caledonian Journal of Engineering, 3(2) 1-5248*