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AN ALGORITHM FOR SOLVING A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM

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Abstract: *In this paper, a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. An algorithm to find the efficient cost-time trade off pairs in a capacitated fixed charge bi-criterion indefinite quadratic transportation problem is developed. The algorithm is based on the concept of solving the indefinite quadratic fixed charge transportation problem and reading the corresponding time from the time matrix. It is illustrated with the help of a numerical example.*

Keywords: *optimum time cost trade off , capacitated transportation problem, fixed charge, bi-criterion indefinite quadratic transportation problem.*

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1. INTRODUCTION:

In the classical transportation problem, the cost of transportation is directly proportional to the number of units of the commodity transported. But in real world situations, when a commodity is transported , a fixed cost is incurred in the objective function .The fixed cost may represent the cost of renting a vehicle, landing fees at an airport, set up cost for machines etc. The fixed charge transportation problem was originally formulated by G.B Dantzig and W. Hirisch [13] in 1954. After that , several procedures for solving fixed charge transportation problems were developed.

Sometimes, there may exist emergency situations such as fire services, ambulance services, police services etc when the time of transportation is more important than cost of transportation. Hammer[12], Szwarc[14], Garfinkel et al. [7], Bhatia et al. [5] and many others have studied the time minimizing transportation problem which is a special case of bottleneck linear programming problems.

Another important class of transportation problem consists of capacitated transportation problem. Many researchers like Gupta et.al. [8-11], Dahiya et.al. [6] have contributed in this field. In 1976, Bhatia et .al. [4] provided the time cost trade off pairs in a linear transportation problem. Then in 1994, Basu et.al. [3] developed an algorithm for finding the optimum time cost trade off pairs in a fixed charge linear transportation problem giving same priority to cost and time.

Another class of transportation problems , where the objective function to be optimized is the product of two linear functions which gives more insight in to the situation than the optimization of each criterion. Arora et. al. [1-2] have contributed in the field of indefinite quadratic transportation problem.

In this paper, a capacitated fixed charge indefinite quadratic transportation problem giving same priority to cost and time is studied. An algorithm to identify the efficient cost time trade off pairs for the problem is developed.

2. MATHEMATICAL MODEL FOR A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM:

(P1): $$\min\left\{\left(\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}\right)+\sum_{i\in I} F_i, \max_{i\in I, j\in J}\left(t_{ij} / x_{ij}>0\right)\right\}$$

subject to

$$\sum_{j\in J} x_{ij} \le a_i; \forall i \in I$$

(1)

$$\sum_{i\in I} x_{ij} = b_j; \forall j \in J$$

(2)

$$l_{ij} \le x_{ij} \le u_{ij}; \forall (i, j) \in I \times J \quad (3)$$

I = {1, 2, … m} is the index set of m origins.

J = {1, 2, …, n} is the index set of n destinations.

x_{ij} = number of units transported from origin i to the destination j.

c_{ij} = variable cost of transporting one unit of commodity from i^{th} origin to the j^{th} destination.

d_{ij} = the per unit damage cost or depreciation cost of commodity transported from i^{th} origin to the j^{th} destination.

l_{ij} and u_{ij} are the bounds on number of units to be transported from i^{th} origin to j^{th} destination.

t_{ij} is the time of transporting goods from i^{th} origin to the j^{th} destination.

F_i is the fixed cost associated with i^{th} origin. The fixed cost F_i depends upon the amount supplied from the i^{th} origin to different destinations and is defined as follows.

$$F_i = \sum_{l=1}^{p} F_{il}\delta_{il} \quad , i=1, 2, 3\ldots.m \quad , l=1,2,3\ldots p$$

where $$\delta_{il} = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} x_{ij} > a_{il} \\ 0 & \text{otherwise} \end{cases}$$ for l=1,2,3……..p, i=1,2,……m

Here, 0 = $a_{i1} < a_{i2} \ldots < a_{ip}$ .Also a_{i1} , a_{i2} … , a_{ip} (i = 1,2, … m) are constants and F_{il} are the fixed costs $\forall$ i= 1, 2 …m, l = 1,2 ..p

In the problem (P1), we need to minimize the total transportation cost and depreciation cost simultaneously to be transported from the i^{th} origin to the j^{th} destination Also we need to find the different cost - time trade off pairs.

3. THEORETICAL DEVELOPMENT:

Since $\sum_{i\in I} a_i > \sum_{j\in J} b_j$, the given problem is not a balanced problem . Therefore we introduce a dummy destination (n+1) with demand $b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ .The cost and time allocated in this $(n+1)^{th}$ column is zero. The resulting balanced problem is given below.

$$(P1'): \min\left\{\left(\sum_{i\in I}\sum_{j\in J'} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J'} d_{ij}x_{ij}\right)+\sum_{i\in I} F_i, \max_{i\in I, j\in J'}\left(t_{ij} / x_{ij} > 0\right)\right\}$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i; \forall i \in I \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j; \forall j \in J' \tag{5}$$

$$l_{ij} \le x_{ij} \le u_{ij}; \forall i \in I, j \in J \tag{6}$$

where J´={1,2,3……n,n+1}and $x_{i,n+1} \ge 0; l_{i,n+1} = 0; u_{i,n+1} \ge 0; \forall i \in I$

$$c_{i,n+1} = 0 = d_{i,n+1} = t_{i,n+1}; \forall i \in I$$

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

F_i for i=1,2,…m are defined as in problem (P1).

In order to solve the problem (P1´), we separate it in to two problems (P2) and (P3) where

(P2): minimize the cost function $\left\{\left(\sum_{i\in I}\sum_{j\in J'} c_{ij}x_{ij}\right)\left(\sum_{i\in I}\sum_{j\in J'} d_{ij}x_{ij}\right)+\sum_{i\in I} F_i\right\}$ subject to (4), (5) and (6) .

(P3): minimize the time function $\left\{\max_{i\in I, j\in J'}\left(t_{ij} / x_{ij} > 0\right)\right\}$ subject to (4), (5) and (6).

To obtain the set of efficient cost- time trade off pairs, we first solve the problem (P2) and read the time with respect to the minimum cost Z where time T is given by the problem (P3) .

At the first iteration , let Z_1^* be the minimum total cost of the problem (P2).Find all alternate solutions i.e. solutions having the same value of $Z = Z_1^*$. Let these solutions be $X_1, X_2, \ldots X_n$. Corresponding to these solutions, find the time $T_1^* = \min_{X_1, X_2 \ldots X_n} \left\{ \max_{i \in I, j \in J'} \left(t_{ij} / x_{ij} > 0 \right) \right\}$.

Then (Z_1^*, T_1^*) is called the first cost time trade off pair.Modify the cost with respect to the time so obtained i.e. define $c_{ij} = \begin{Bmatrix} M & \text{if } t_{ij} \ge T^* \\ c_{ij} & \text{if } t_{ij} < T^* \end{Bmatrix}$ and form the new problem (P2′) and find its optimal solution and all feasible alternate solutions. Let the new value of Z be Z_2^* and the corresponding time is T_2^* , then (Z_2^*, T_2^*) is the second cost time trade off pair. Repeat this process. Suppose that after q th iteration ,the problem becomes infeasible. Thus, we get the following complete set of cost- time trade off pairs. (Z_1^*, T_1^*) , (Z_2^*, T_2^*),(Z_3^*, T_3^*),................(Z_q^*, T_q^*) where $Z_1^* \le Z_2^* \le Z_3^* \le \ldots \le Z_q^*$ and $T_1^* > T_2^* > T_3^* \ldots > T_q^*$.The pairs so obtained are pareto optimal solution of the given problem. Then we identify the minimum cost Z_1^* and minimum time T_q^* among the above trade off pairs. The pair (Z_1^*, T_q^*) with minimum cost and minimum time is termed as the ideal pair which can not be achieved in practical situations.

Theorem 1: Let $X = \{X_{ij}\}$ be a basic feasible solution of problem (P2) with basis matrix B. Then it will be an optimal basic feasible solution if

$$R_{ij}^1 = \theta_{ij}\left[z_1(d_{ij}-z_{ij}^2)+z_2(c_{ij}-z_{ij}^1)+\theta_{ij}(c_{ij}-z_{ij}^1)(d_{ij}-z_{ij}^2)\right] + \Delta F_{ij} \ge 0; \forall (i,j) \in N_1$$

and

$$R_{ij}^2 = \theta_{ij}\left[\theta_{ij}(c_{ij}-z_{ij}^1)(d_{ij}-z_{ij}^2)-z_1(d_{ij}-z_{ij}^2)-z_2(c_{ij}-z_{ij}^1)\right] + \Delta F_{ij} \ge 0; \forall (i,j) \in N_2$$

such that

$$u_i^1 + v_j^1 = c_{ij} \quad \forall (i,j) \in B \tag{7}$$

$$u_i^2 + v_j^2 = d_{ij} \quad \forall (i,j) \in B \tag{8}$$

$$u_i^1 + v_j^1 = z_{ij}^1 \quad \forall (i,j) \in N_1 \text{ and } N_2 \tag{9}$$

$$u_i^2 + v_j^2 = z_{ij}^2 \quad \forall (i,j) \in N_1 \text{ and } N_2 \tag{10}$$

ΔF_{ij} is the change in fixed cost $\sum_{i \in I} F_i$ when some non basic variable x_{ij} undergoes change by an amount of θ_{ij}.

Z_1 = value of $\sum_{i \in I}\sum_{j \in J} c_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

Z_2 = value of $\sum_{i \in I}\sum_{j \in J} d_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: $u_i^1, v_j^1, u_i^2, v_j^2$ are the dual variables which are determined by using equations (7) to (10) and taking one of the u_i's or v_j's. as zero.

Proof: Let Z^0 be the objective function value of the problem (P2).

Let $z^0 = Z_1Z_2 + F^0$ where $F^0 = \sum_{i \in I} F_i$

Let $\hat{z}$ be the objective function value at the current basic feasible solution $\hat{X} = \{x_{ij}\}$ corresponding to the basis B obtained on entering the non basic cell $x_{ij} \in N_1$ in to the basis which undergoes change by an amount θ_{ij} and is given by min{$u_{ij} - l_{ij}$; $x_{ij} - l_{ij}$ for all basic cells (i,j) with a (- θ)entry in the θ-loop; $u_{ij} - x_{ij}$ for all basic cells (i,j) with a (+ θ)entry in the θ-loop}.

Then $\hat{z} = \left[z_1 + \theta_{ij}\left(c_{ij} - z_{ij}^1\right)\right]\left[z_2 + \theta_{ij}\left(d_{ij} - z_{ij}^2\right)\right] + F^0 + \Delta F_{ij}$

$$\hat{z} - z^0 = \left[z_1z_2 + \theta_{ij}z_1(d_{ij} - z_{ij}^2) + z_2\theta_{ij}(c_{ij} - z_{ij}^1) + \theta_{ij}^2(c_{ij} - z_{ij}^1)(d_{ij} - z_{ij}^2) - z_1z_2\right] + \Delta F_{ij}$$

$$= \theta_{ij}\left[z_1(d_{ij} - z_{ij}^2) + z_2(c_{ij} - z_{ij}^1) + \theta_{ij}(c_{ij} - z_{ij}^1)(d_{ij} - z_{ij}^2)\right] + \Delta F_{ij}$$

This basic feasible solution will give an improved value of z if $\hat{z} < z^0$.It means

If $\theta_{ij}\left[z_1(d_{ij} - z_{ij}^2) + z_2(c_{ij} - z_{ij}^1) + \theta_{ij}(c_{ij} - z_{ij}^1)(d_{ij} - z_{ij}^2)\right] + \Delta F_{ij} < 0$ (11)

Therefore one can move from one basic feasible solution to another basic feasible solution on entering the cell (i,j) $\in N_1$ in to the basis for which condition (11) is satisfied.

It will be an optimal basic feasible solution if

$$R_{ij}^1 = \theta_{ij}\left[z_1(d_{ij} - z_{ij}^2) + z_2(c_{ij} - z_{ij}^1) + \theta_{ij}(c_{ij} - z_{ij}^1)(d_{ij} - z_{ij}^2)\right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1$$

Similarly, when non basic variable $x_{ij} \in N_2$ undergoes change by an amount θ_{ij} then

$$\hat{z} - z^0 = \theta_{ij}\left[\theta_{ij}(c_{ij}-z^1_{ij})(d_{ij}-z^2_{ij})-z_1(d_{ij}-z^2_{ij})-z_2(c_{ij}-z^1_{ij})\right] + \Delta F_{ij} < 0$$

It will be an optimal basic feasible solution if

$$R^2_{ij} = \theta_{ij}\left[\theta_{ij}(c_{ij}-z^1_{ij})(d_{ij}-z^2_{ij})-z_1(d_{ij}-z^2_{ij})-z_2(c_{ij}-z^1_{ij})\right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_2$$

4. ALGORITHM:

Step1: Given a capacitated fixed charge bi-criterion indefinite quadratic transportation problem (P1). Introduce a dummy destination to form the related balanced transportation problem (P1´) and then separate it in to two problems (P2) and (P3). Find a basic feasible solution of problem (P2) with respect to variable cost only. Let B be its corresponding basis.

Step 2: Calculate the fixed cost of the current basic feasible solution and denote it by F(current)

where F(current) = $\sum_{i=1}^{m} F_i$

Step 3(a): Find $\Delta F_{ij} = F(NB) - F(current)$ where F(NB) is the total fixed cost obtained when some non basic cell (i,j) undergoes change.

Step 3(b): Calculate θ_{ij} ,(c_{ij}-z^1_{ij}) , ($d_{ij} - z^2_{ij}$), z_1, z_2 for all non basic cells such that

$$u^1_i + v^1_j = c_{ij} \quad \forall (i,j) \in B$$

$$u^2_i + v^2_j = d_{ij} \quad \forall (i,j) \in B$$

$$u^1_i + v^1_j = z^1_{ij} \quad \forall (i,j) \in N_1 \text{ and } N_2$$

$$u^2_i + v^2_j = z^2_{ij} \quad \forall (i,j) \in N_1 \text{ and } N_2$$

Z_1 = value of $\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

Z_2 = value of $\sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis B.

θ_{ij} = level at which a non basic cell (i,j) enters the basis replacing some basic cell of B.

N_1 and N_2 denotes the set of non basic cells (i,j) which are at their lower bounds and upper bounds respectively.

Note: $u^1_i, v^1_j, u^2_i, v^2_j$ are the dual variables which are determined by using above equations and taking one of the u_i's or v_j's as zero.

Step 3(c) : Find $R^1_{ij} \forall (i, j) \in N_1$ and $R^2_{ij} \forall (i, j) \in N_2$ where

$$R^1_{ij} = \theta_{ij}\left[z_1(d_{ij}-z^2_{ij})+z_2(c_{ij}-z^1_{ij})+\theta_{ij}(c_{ij}-z^1_{ij})(d_{ij}-z^2_{ij})\right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1 \text{ and}$$

$$R^2_{ij} = \theta_{ij}\left[\theta_{ij}(c_{ij}-z^1_{ij})(d_{ij}-z^2_{ij})-z_1(d_{ij}-z^2_{ij})-z_2(c_{ij}-z^1_{ij})\right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2$$

Step 4: If $R^1_{ij} \geq 0 \forall (i, j) \in N_1$ and $R^2_{ij} \geq 0 \forall (i, j) \in N_2$ then the current solution so obtained is the optimal solution to (P2).Go to step 5.Otherwise, some (i,j) $\in N_1$ for which $R^1_{ij} < 0$ or some (i,j) $\in N_2$ for which $R^2_{ij} < 0$ will undergo change. Go to step 2.

Step 5: Let Z^1 be the optimal cost of (P2´) yielded by the basic feasible solution $\{y'_{ij}\}$. Find all alternate solutions to the problem (P2) with the same value of the objective function. Let these solutions be $X_1, X_2, \ldots X_n$ and $T_1 = \min_{X_1, X_2 \ldots X_n} \left\{ \max_{i \in I, j \in J'} \left(t_{ij} / x_{ij} > 0 \right) \right\}$.Then the corresponding pair (Z^1 **,** T^1) will be the first time cost trade off pair for the problem (P1).To find the second cost- time trade off pair, go to step 6.

Step6: Define $c^1_{ij} = \begin{cases} M & \text{if } t_{ij} \geq T^1 \\ c_{ij} & \text{if } t_{ij} < T^1 \end{cases}$

where M is a sufficiently large positive number. Form the corresponding capacitated fixed charge quadratic transportation problem with variable cost c^1_{ij}. Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P1) at the end of q^{th} iteration are given by (Z^1, T^1), (Z^2,T^2)……….(Z^q , T^q) where $Z^1 \leq Z^2 \leq \ldots \leq Z^q$ and $T^1 > T^2 > \ldots > T^q$.

Remark 2: The pair (Z^1, T^q) with minimum cost and minimum time is the ideal pair which can not be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5. NUMERICAL ILLUSTRATION:

Consider a 3 x 3 capacitated fixed charge bi-criterion indefinite quadratic transportation problem. Table 1 gives the values of c_{ij}, d_{ij}, a_i ,b_j for i=1,2,3 and j=1,2,3

Table 1:Cost matrix of problem (P1)

	D_1	D_2	D_3	a_i
O_1	5 4	9 2	9 1	30
O_2	4 3	6 7	2 4	40
O_3	2 2	1 9	1 4	80
b_j	30	20	30	

Note: values in the upper left corners are c_{ij} ’s and values in lower left corners are d_{ij} ’s for i=1,2,3.and j=1,2,3.

$1\le x_{11} \le 10$, $2 \le x_{12} \le 10$, $0 \le x_{13} \le 5$,$0\le x_{21} \le 15$, $3 \le x_{22} \le 15$, $1 \le x_{23} \le 20$,$0\le x_{31}\le 20$,

$0\le x_{32}\le 13$, $0\le x_{33}\le 25$

$F_{11}= 100$, $F_{12} = 50$, $F_{13} = 50$, $F_{21} = 150$, $F_{22} = 100$, $F_{23} = 50$,$F_{31}= 200$, $F_{32}= 150$, $F_{33} = 100$

$F_i = \sum_{l=1}^{3} F_{il}\delta_{il}$ for l=1,2,3 where for i= 1,2,3

$$\delta_{i1} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 10 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{i3} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 20 \\ 0 & \text{otherwise} \end{cases}$$

Table 2 gives the values of t_{ij}’s for i=1,2,3 and j=1,2,3

Table 2 :values of t_{ij}

	D_1	D_2	D_3
O_1	15	8	13
O_2	10	13	11

O_3	12	10	9

Introduce a dummy destination in Table 1 with $c_{i4} = 0 = d_{i4}$ for all i = 1,2 ,3 .and b_4 =70

Now we find an initial basic feasible solution of problem (P2) which is given in table 3 below.

Table 3: A basic feasible solution of problem (P2)

	D_1	D_2	D_3	D_4	u_i^1	u_i^2	F(current)
O_1	5 $\underline{1}$ 4	9 $\underline{2}$ 2	9 $\underline{0}$ 1	0 **27** 0	0	0	100
O_2	4 **9** 3	6 **5** 7	2 **5** 4	0 **21** 0	0	0	250
O_3	2 2 $\overline{20}$	1 9 $\overline{13}$	1 4 $\overline{25}$	0 **22** 0	0	0	450
v_j^1	4	6	2	0			
v_j^2	3	7	4	0			

Note: entries of the form $\underline{a}$ and $\overline{b}$ represent non basic cells which are at their lower and upper bounds respectively. Entries in bold are basic cells.

F(current) = 800, z_1 = 177 , z_2 = 347

Table 4: Calculation of optimality condition

NB	O_1D_1	O_1D_2	O_1D_3	O_3D_1	O_3D_2	O_3D_3
θ_{ij}	9	2	4	6	10	15
$c_{ij} - z_{ij}^1$	1	3	7	-2	-5	-1
$d_{ij} - z_{ij}^2$	1	-5	-3	-1	2	0
F(NB)	750	800	800	850	850	850
ΔF_{ij}	-50	0	0	50	50	50
R_{ij}^1	4747	252	7256			
R_{ij}^2				5348	12860	5255

Since $R^1_{ij} \geq 0 \ \forall(i, j) \in N_1$ and $R^2_{ij} \geq 0; \forall(i, j) \in N_2$, the solution in table 3 is an optimal solution of (P2). Therefore minimum cost = Z^1 = (177 x 347) + 800 = 62219 and corresponding time is T^1 = 15 . Hence the first time cost trade off pair is (62219,15)

Define $c^1_{ij} = \begin{Bmatrix} M & \text{if } t_{ij} \geq T^1 = 15 \\ c_{ij} & \text{if } t_{ij} < T^1 = 15 \end{Bmatrix}$ and solving the given problem , we get the second cost time trade off pair as (62219,13). Similarly third cost - time trade off pair is (62471, 12). After that the problem becomes infeasible. Hence the cost time trade off pairs are (62219, 15) , (62219,13),(62471,12).

6. CONCLUSION

In order to solve a capacitated fixed charge bi – criterion indefinite quadratic transportation problem , we separate the problem in to two problems. One of them being an indefinite quadratic programming problem has its optimal solution at its extreme point. After finding the optimal solution we read the time from the time matrix corresponding to $x_{ij} > 0$. Then we define the new cost and new problem with these costs to find the second cost - time trade off pair. The process is repeated till the problem becomes infeasible. The process must end after a finite number of steps because our algorithm moves from one extreme point to another which is finite in number.

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