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KINEMATICS OF AN OVERCONSTRAINED MECHANISM IN PRACTICE

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Abstract: *In 1939 Paul Schatz, a Swiss anthroposophist and geometrician had invented a mechanism which with few links generates spatial motions. Mixing machine based on this Schatz mechanism uses a highly ordered form of three dimensional motion that brings centrifugal and centripetal forces to dynamic balance. This results in ideal mixing environment. While exploring this mechanism, it was then clear that this mechanism is a six link spatial mechanism which does not satisfy the Kutzbach criterion of full cycle mobility. Such practical application of overconstrained mechanism has very interesting characteristics. Without the use of cams or gears it performs spatial motion. The work described in this paper is an attempt to describe general kinematics of this six link mechanism involving only revolutes, using matrix method. As discussed here, a spatial mechanism containing only revolutes is derived from the seven link kinematics chain. However mechanism described here has only six links including the fixed base. If the link lengths are selected arbitrary, then the mechanism will result in immobility with no degrees of freedom. Mobility of the mechanism is because of specific linear dimensions only. The mobility of the mechanism is proved with this kinematics description.*

Keywords: *overconstrained mechanism, 3-D Mixer, D & H Matrix, kinematics, mobility criterion, spatial mechanism*

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OVERCONSTRAINED MECHANISM:

A mechanism is commonly identified as a set of moving or working parts in a machine or other device essentially as a means of transmitting, controlling, or constraining relative movement. A mechanism is often assembled from gears, cams and linkages, though it may contain other specialized components, such as springs, ratchets, brakes, and clutches, as well.

Reuleaux published the first book on theoretical kinematics of mechanisms in 1875 (Hunt, 1978). Later on the general mobility criterion of an assembly was established by Grübler in 1921 and Kutzbach in 1929, respectively (Phillips, 1984), based on the topology of the assembly [1]. However, it was found that this criterion is not a necessary condition. Some specific geometric condition in an assembly could make it a mechanism even though it does not obey the mobility criterion. This type of mechanisms is called an over-constrained mechanism.

The first published research on over-constrained mechanisms can be traced back to 150 years ago when Sarrus discovered a six-bar mechanism capable of rectilinear motion [1]. Gradually more over-constrained mechanisms were discovered by other researchers in the next half a century. However, most over-constrained mechanisms have rarely been used in industrial applications because of the development of gears, cams and other means of transmission, except two of them: the double- Hooke's-joint linkage, which is widely applied as a transmission coupling, and the Schatz linkage, which is used as a 3-Dimensional Mixing machine for mixing fluids and powders. Over the recent half century, very few Over-constrained mechanisms have been found. Most research work on Over-constrained mechanisms is mainly focused on their kinematic characteristics.

LINKAGES AND OVER-CONSTRAINED LINKAGES [1]

A linkage is a particular type of mechanism consisting of a number of interconnected components, individually called links. The physical connection between two links is called a joint. All joints of linkages are lower pairs, i.e. surface-contact pairs, which include spherical joints, planar joints, cylindrical joints, revolute joints, prismatic joints, and screw joints. Here we limit our attention to linkages whose links form a single loop and are connected only by revolute joints, also called rotary hinges. These joints allow one-degree-of-freedom movement between the two links that they connect. The kinematic variable for a revolute joint is the angle measured around the two links that it connects.

From classical mobility analysis of mechanisms, it is known that the mobility m of a linkage composed of n links that are connected with p joints can be determined by the Kutzbach (or Grübler) mobility criterion (Hunt, 1978):

$$m = 6(n - p - 1) + \sum f \quad \text{...} \quad (1)$$

where Σ f is the sum of kinematic variables in the mechanism.

For an n--link closed loop linkage with revolute joints, p = n, and the kinematic variable Σ f = n. Then the mobility criterion in (1) becomes;

$$m = n - 6 \quad \text{...} \quad (2)$$

So in general, to obtain a mobility of one, a linkage with revolute joints needs at least seven links. It is important to note that (2) is not a necessary condition because it considers only the topology of the assembly. There are linkages with full-range mobility even though they do not meet the mobility criterion. These linkages are called over-constrained linkages. Their mobility is due to the existence of special geometry conditions among the links and joint axes that are called over-constrained conditions.

Denavit and Hartenberg (1966) set forth a standard approach to the analysis of linkages, where the geometric conditions are taken into account. They pointed out that, for a closed loop in a linkage, the necessary and sufficient mobility condition is that the product of the transform matrices equals the unit matrix, i.e.

$$[T_{n1}]\ldots\ldots [T_{34}][T_{23}][T_{12}] = [I] \quad \text{................................} \quad (3)$$

Where $[T_{i(i+1)}]$ is the transfer matrix between the system of link (i −1)i and the system of link i(i +1).

Over-constrained mechanisms have many appealing characteristics. Most of them are spatial mechanisms. Their spatial kinematic characteristics make them good candidates in modern linkage designs where spatial motion is needed. Another advantage of Over-constrained mechanisms is that they are mobile using fewer links and joints than it is expected. For example, in normal closed loop revolute joint spatial mechanisms, the linkage should have at least seven links to be mobile. Over-constrained mechanisms can be mobile with four, five or six links. Fewer links and joints in a mechanism mean reduction in cost and complexity [2].

While many over-constrained mechanisms have been discovered, only a few of them have been used in practical applications. There are many reasons for this. Most of the engineers

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are unaware of the existence of spatial over-constrained mechanisms and their properties. For example, very few engineers in industry know the four-bar Bennett or the six-bar Bricard spatial mechanisms and their properties and hence they can not consider these mechanisms in their designs. The other reason for not using over-constrained mechanisms in industrial applications is that most of the known over-constrained mechanisms have complex kinematic properties. This is because these mechanisms have been found using mobility criteria only and not other criteria as well, such as to satisfy a desired input-output relationship.

The minimum number of links to construct a mobile loop with revolute joints is four as a loop with three links and three revolute joints is either a rigid structure or an infinitesimal mechanism when all three revolute axes are coplanar and intersect at a single point (Phillips, 1990). So, 3D over-constrained linkages can have four, five or six links. When these linkages consist of only revolute joints, they are called 4R, 5R or 6R linkages.

SCHATZ LINKAGE[1]

The Schatz linkage discovered and patented by Schatz was derived from a special trihedral Bricard linkage (Phillips, 1990). First, set this trihedral Bricard linkage in a configuration such that angles between the adjacent links are all π /2, see Fig 1.Then replace links 61, 12, and 56 with a new link 61 of zero twist and a new pair of parallel shafts 12 and 56 (two links of zero length). By now, a new asymmetrical 6R linkage has been obtained with single degree of mobility. The dimension constraints of the linkage are as follows:

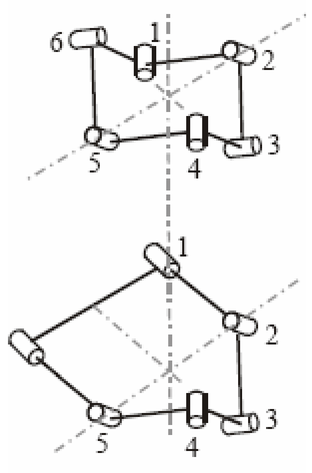

Figure 1. P Schatz Linkages

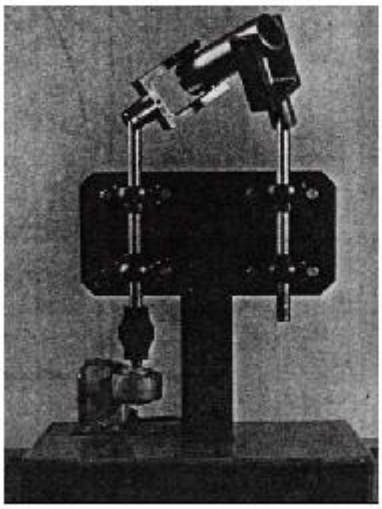

Figure 2. Schatz 3D Mixer

$$a_{12} = a_{56} = 0,\ a_{23} = a_{34} = a_{45} = a,\ a_{61} = \sqrt{3}a \quad \text{...} (4)$$

$$\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{45} = \alpha_{56} = \pi/2 \quad \text{...} (5)$$

$$R_1 = -R_6,\quad R_2 = R_3 = R_4 = R_5 = 0 \quad \text{..} (6)$$

Brát (1969) studied the kinematic description of the Schatz linkage using the matrix method. This linkage is also known by the name Turbula because it constitutes the essential mechanism of a machine by that name, see Fig. 2. It is used for mixing fluids and powders.

Spatial mechanisms have little applications, because the complexities of kinematic relations have prevented solutions by conventional methods. The graphical method has major limitations in their applications to spatial problems as they depend on the choices which require a visualization of the motion & visualization of spatial mechanism is very difficult.

A method based on the matrix algebra, initiated by J. Denavit & R. S. Hartenberg is proved very useful till today. The procedure defines all parameters required for a kinematic analysis and allows formulating spatial problems in terms of matrix equations

DESCRIPTION OF D & H MATRIX ALGEBRA METHOD FOR ANALYSIS OF SPATIAL MECHANISMS [3]

x_i = axis formed by common perpendicular directed from z_{i-1} to z_i. If these axes intersect, orientation of x_i is arbitrary.

y_i = axis implicitly defined to form a right handed Cartesian coordinate system, x_i, y_i, z_i

a_i = length of common perpendicular from z_i to z_{i+1}; always positive

α_i = angle from positive z_i to positive z_{i+1}, measured counterclockwise about positive X_i

In any simple closed chain of binary links, the Grubler criterion requires that, for constrained motion, the sum of the degrees of freedom of the individual joints be equal to seven. Even with the exceptions produced by redundant constraints, this implies that the total number of links, “n” must be less than or equal to seven.

$$n \le 7$$

If the dimensions of a linkage are measured according to the format outlined here, the geometry of any link i & its position relative to i-1 may be completely specified by four parameters, a_i, α_i, θ_i and s_i as shown in figure 3.

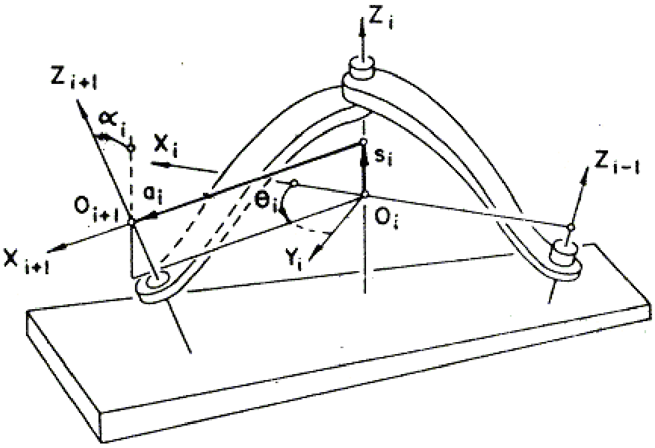

Figure 3. D & H Symbolic Notations

These four parameters should be measured for each joint or pair of the linkage according to the following set of conventions:

i = number of particular joint or pair. Input pair will be taken as 1, and remaining joints will be numbered consequently around closed loop.

z_i = characteristic axis of rotation for pair involved. All the z – axis should be defined clearly and arbitrary orientation indicated.

x_{i+1}

θ_i = angle from positive x_i to positive x_{i+1} measured counterclockwise about positive z_i

s_i = distance along z_i from x_i to x_{i+1}. Takes sign from orientation of positive z_i

Once the four parameters have been established for each pair of a linkage, the geometry of the linkage is completely specified, and can be represented by a symbolic equation of the form a_i, α_i, θ_i and s_i

$$R_1\begin{pmatrix}0\\ \frac{\pi}{2}\\ \theta_1\\ s_1\end{pmatrix}R_2\begin{pmatrix}a_2\\ -\frac{\pi}{2}\\ \frac{\pi}{2}\\ 0\end{pmatrix}R_3\begin{pmatrix}a_3\\ \frac{\pi}{2}\\ \frac{\pi}{2}\\ 0\end{pmatrix}R_4\begin{pmatrix}a_4\\ -\frac{\pi}{2}\\ 0\\ 0\end{pmatrix}R_5\begin{pmatrix}0\\ \frac{\pi}{2}\\ \theta_5\\ 0\end{pmatrix}R_6\begin{pmatrix}a_6\\ 0\\ \pi\\ s_6\end{pmatrix}=I \quad \text{.............. (7)}$$

Where R_1 denotes that pair 1 is a revolute joint with θ_1 as variable, P_2 denotes that pair 2 is a prismatic pair with variable s_2, and so on. I indicate that the chain is closed. In addition, it will always be assumed that pair 1 is the input, so that θ_1, is the input variable of the linkage. Although equation 7 is only a symbolic equation describing the geometry of the mechanism, it does lead to a convenient matrix equation. The x_i, y_i, z_i axes define a right handed Cartesian-coordinate system rigidly attached to link i. The four parameters, a_i, α_i, θ_i and s_i, fix the position of the coordinate system of link i +1 relative to that of link i. The relative positions of these two coordinate systems can be stated analytically in terms of a (4 X 4) transformation matrix involving only the four parameters, a_i, α_i, θ_i and s_i,

$$A_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_i \cos\theta_i & \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i \\ a_i \sin\theta_i & \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i \\ s_i & 0 & \sin\alpha_i & \cos\alpha_i \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

Since the mechanism is simple closed loop, the n + 1 coordinate system is identical with coordinate system 1. This may be expressed in terms of the matrices as,

$$A_1 \cdot A_2 \cdots\cdots A_n = I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9),$$

where n is the number of links and I is the (4 X 4) unit matrix.

This equation completely describes the geometry of the linkage, and its solutions should yield a complete displacement analysis, i. e., the values of all pair variables of the linkage in terms of the fixed parameters and the input variable θ_1 (or s_1).

KINEMATIC DESCRIPTION OF 3-D MIXER

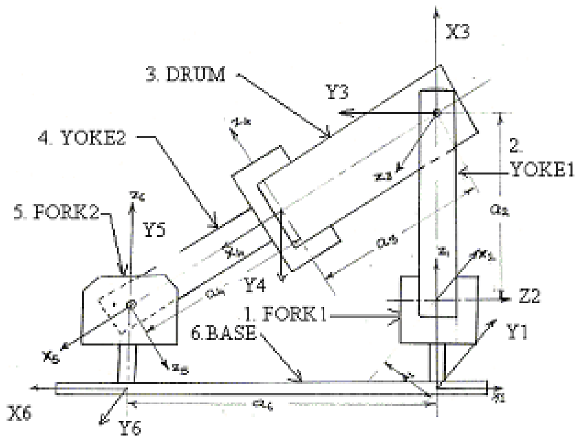

Figure 4. 6R 3D Mixer Linkages Showing D & H Notations

The 3-D mixing mechanism mentioned here comprises a six link spatial mechanism which involves only revolute joints.

The kinematic schematic of this mechanism is shown in figure 4. The main member of the mechanism is the “Drum” (3), which supports the container with material to be mixed. The drum moves with general spatial motion. The driving member is "Fork" (1) which is same as member (5). Member (2) and (4) are named as "Yoke" forms the universal joint with drum, member (3). Member (6) is the fixed base, on which the whole assembly of drum, yokes and forks is mounted. Attempt is made here to derive the kinematic model of this mechanism following the Denavit & Hartenberg matrix method.

Characteristic axis of motion at the joint of member (1)-fork1 with member (6)-Fixed base and characteristic axis of motion at the joint of member (5)-fork2 with member (6)-fixed base are parallel, (Z_1 II Z6). Characteristic axis of motion at the joint of member (1)-fork1 with member (2)-yoke1 and characteristic axis of motion at the joint of member (5)-fork2 with member (4)-yoke2, are initially at right angle and keeps on changing their position while mechanism is in motion, (Axis Z_5 and Axis Z_6). Characteristic axis of motion at the joint of member (2)-yoke1 with member (3)-Cylinder and the characteristic axis of motion at the joint of member (4)-yoke2 and member (3)-cylinder are always at right angle, ($Z_3 \perp$ Z4).

Ordinarily the main members of the mechanism perform only rotational and translatory motion while the spatial character of the mechanism is caused by connecting members. This is the case, for example in a Hooke's joint with the crossed or skewed input and output shafts.

The coordinate systems at the joint position of each members are assigned according to Denavit & Hartenberg procedure and four parameters (a_i, α_i, θ_i and s_i,) have been assigned for each pair of a linkage to specify the geometry of any link i and its position relative to link i – 1 as shown in figure 4.

Table 1. D & H Parameters of links

i	1	2	3	4	5	6
a_i	0	a_2	a_3	a_4	0	a_6
α_i	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	0
s_i	s_1	0	0	0	0	s_6
θ_i	θ_1	$\frac{\pi}{2}$	$\frac{\pi}{2}$	0	θ_5	π

Now the geometry of linkage is completely specified and can be represented by a symbolic equation of the form:

$$R_1\begin{pmatrix}0\\ \frac{\pi}{2}\\ \theta_1\\ s_1\end{pmatrix}R_2\begin{pmatrix}a_2\\ -\frac{\pi}{2}\\ \frac{\pi}{2}\\ 0\end{pmatrix}R_3\begin{pmatrix}a_3\\ \frac{\pi}{2}\\ \frac{\pi}{2}\\ 0\end{pmatrix}R_4\begin{pmatrix}a_4\\ -\frac{\pi}{2}\\ 0\\ 0\end{pmatrix}R_5\begin{pmatrix}0\\ \frac{\pi}{2}\\ \theta_5\\ 0\end{pmatrix}R_6\begin{pmatrix}a_6\\ 0\\ \pi\\ s_6\end{pmatrix}=I \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Where R_i denotes that pair i is revolute pair with θ_i as variable. I indicate that the chain is closed. In addition it is clear that pair 1 is the input pair, so that θ_1 is the input variable of the linkage. The homogeneous transformation matrices relating a link with the adjoining link are as follows:

$$A_1=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta_1 & 0 & \sin\theta_1\\ 0 & \sin\theta_1 & 0 & -\cos\theta_1\\ s_1 & 0 & 1 & 0\end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$A_2=\begin{pmatrix}1 & 0 & 0 & 0\\ a_2\cos\theta_2 & \cos\theta_2 & 0 & -\sin\theta_2\\ a_2\sin\theta_2 & \sin\theta_2 & 0 & \cos\theta_2\\ 0 & 0 & -1 & 0\end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$A_3=\begin{pmatrix}1 & 0 & 0 & 0\\ a_3\cos\theta_3 & \cos\theta_3 & 0 & \sin\theta_3\\ a_3\sin\theta_3 & \sin\theta_3 & 0 & -\cos\theta_3\\ 0 & 0 & 1 & 0\end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$A_4=\begin{pmatrix}1 & 0 & 0 & 0\\ a_4\cos\theta_4 & \cos\theta_4 & 0 & -\sin\theta_4\\ a_4\sin\theta_4 & \sin\theta_4 & 0 & \cos\theta_4\\ 0 & 0 & -1 & 0\end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$A_5=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta_5 & 0 & \sin\theta_5\\ 0 & \sin\theta_5 & 0 & -\cos\theta_5\\ 0 & 0 & 1 & 0\end{pmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

$$A_6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a_6 \cos\theta_6 & \cos\theta_6 & -\sin\theta_6 & 0 \\ a_6 \sin\theta_6 & \sin\theta_6 & \cos\theta_6 & 0 \\ d_6 & 0 & 0 & 1 \end{pmatrix} \quad \text{.. (16)}$$

And,

$$A_1 A_2 A_3 A_4 A_5 A_6 = I \quad \text{.. (17)}$$

Multiplication of matrices as per equation 17 was done with help of MATLAB. The final product matrix (4X4) contains many complex elements with large terms. This product matrix when equated to identity matrix (4X4), generates twelve scalar equations. Since many equations contain large terms, their further reduction becomes difficult and hence does not result in any feasible solution. Since the mechanism's mobility is already proved by practically manufacturing the mechanism [6], another attempt based on the work of Dr. Vladim'r Brat's **[5]** work was made for the kinematic description of the mechanism.

Kinetic equation of the mechanism can be written as:

$$A_{32}\ A_{21}\ A_{16} = A_{34}\ A_{45}\ A_{56} \quad \text{.. (18)}$$

Where, A_{ij} is the extended transformation matrix of the motion of the link i with respect to link j.

Equation (18) indicates that an arbitrary point of the member (3) (Drum), moves on the same trajectory with respect to the frame regardless of whether the motion of member (3) is described by the member sequence (6), (1), (2), (3) OR (6), (5), (4), (3).

For the individual members coordinate systems (X_i, Y_i, Z_i), i = 1, 2, ….,6 are assigned as shown in figure 4. The transformation matrices for each link with respect to the adjoining link will be as follows;

$$A_{16} = \begin{bmatrix} \cos\varphi_{16} & \sin\varphi_{16} & 0 & 0 \\ -\sin\varphi_{16} & \cos\varphi_{16} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{... (19a)}$$

$$A_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_{21} & \sin\varphi_{21} & 0 \\ 0 & -\sin\varphi_{21} & \cos\varphi_{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{... (19b)}$$

$$A_{32} = \begin{bmatrix} \cos\varphi_{32} & 0 & -\sin\varphi_{32} & 0 \\ 0 & 1 & 0 & 0 \\ \sin\varphi_{32} & 0 & \cos\varphi_{32} & 0 \\ 0 & 0 & a & 1 \end{bmatrix} \quad \text{..} \ (19c)$$

$$A_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_{34} & \sin\varphi_{34} & 0 \\ 0 & -\sin\varphi_{34} & \cos\varphi_{34} & 0 \\ 0 & 0 & a & 1 \end{bmatrix} \quad \text{..........................} \ (19d)$$

$$A_{45} = \begin{bmatrix} \cos\varphi_{45} & 0 & -\sin\varphi_{45} & 0 \\ 0 & 1 & 0 & 0 \\ \sin\varphi_{45} & 0 & \cos\varphi_{45} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{..} \ (19e)$$

$$A_{56} = \begin{bmatrix} \cos\varphi_{56} & \sin\varphi_{56} & 0 & 0 \\ -\sin\varphi_{56} & \cos\varphi_{56} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -a\sqrt{3} & 0 & 1 \end{bmatrix} \quad \text{..} \ (19f)$$

Where:

φ_{16} = rotation of member (1) with respect to member (6) about Z_1

φ_{21} = rotation of member (2) with respect to member (1) about Z_2

φ_{32} = rotation of member (3) with respect to member (2) about Z_3

φ_{34} = rotation of member (3) with respect to member (4) about Z_4

φ_{45} = rotation of member (4) with respect to member (5) about Z_5

φ_{56} = rotation of member (5) with respect to member (6) about Z_6

The configuration shown in figure 4 is the initial position of the mechanism which can also be described as follows:

$\varphi_{16} = 0$ $\qquad$ $\varphi_{21} = \pi/3$

$\varphi_{32} = 0$ $\qquad$ $\varphi_{34} = -2\pi/3$

$\varphi_{45} = 0$ $\qquad$ $\varphi_{56} = 0$

Also the dimensional characteristic of the mechanism is such that the link lengths shown in figure 4 are:

$a_2 = a_3 = a_4 = a$ and $a_6 = \sqrt{3}a$

Substituting the initial position values in equation (19) and substituting equation (19a) to equation (19f) in equation (18) product matrices of the following form is obtained.

$$[a]_{4X4} = [b]_{4X4}$$

By comparing the corresponding terms twelve scalar equations are obtained:

$$\cos\varphi_{32}\cos\varphi_{16} - \sin\varphi_{32}\sin\varphi_{21}\sin\varphi_{16} = \cos\varphi_{45}\cos\varphi_{56} \quad (20)$$

$$\cos\varphi_{32}\sin\varphi_{16} + \sin\varphi_{32}\sin\varphi_{21}\cos\varphi_{16} = \cos_{45}\sin\varphi_{56} \quad (21)$$

$$-\sin\varphi_{32}\cos\varphi_{21} = -\sin\varphi_{45} \quad (22)$$

$$-\cos\varphi_{21}\sin\varphi_{16} = \cos\varphi_{34}\sin\varphi_{56} - \sin\varphi_{34}\sin\varphi_{45}\cos\varphi_{56} \quad (23)$$

$$\cos\varphi_{21}\cos\varphi_{16} = -\cos\varphi_{34}\cos\varphi_{56} - \sin\varphi_{34}\sin\varphi_{45}\sin\varphi_{56} \quad (24)$$

$$\sin\varphi_{21} = -\sin\varphi_{34}\cos\varphi_{45} \quad (25)$$

$$\sin\varphi_{32}\cos\varphi_{16} + \cos\varphi_{32}\sin\varphi_{21}\sin\varphi_{16} = -\sin\varphi_{34}\sin\varphi_{56} - \cos\varphi_{34}\sin\varphi_{45}\cos\varphi_{56} \quad (26)$$

$$\sin\varphi_{32}\sin\varphi_{16} - \cos\varphi_{32}\sin\varphi_{21}\cos\varphi_{16} = \sin\varphi_{34}\cos\varphi_{56} - \cos\varphi_{34}\sin\varphi_{45}\sin\varphi_{56} \quad (27)$$

$$\cos\varphi_{32}\cos\varphi_{21} = -\cos\varphi_{34}\cos\varphi_{45} \quad (28)$$

$$\sin\varphi_{21}\sin\varphi_{16} = \sin\varphi_{45}\cos\varphi_{56} \quad (29)$$

$$\sin\varphi_{21}\cos\varphi_{16} = \sin\varphi_{45}\sin_{56} - \sqrt{3} \quad (30)$$

$$\cos\varphi_{21} = \cos\varphi_{45} \quad (31)$$

Equations (20) to (31) are useful to find out rotation of angles $\varphi_{21,}$ $\varphi_{32,}$ $\varphi_{34,}$ $\varphi_{45,}$ and φ_{56} as a function of input angle φ_{16}. Dependence of the remaining angles upon the input angle φ_{16} is as follows:

$$\sin\varphi_{21} = \left(\sqrt{3}/2\right)\cos\varphi_{16} \quad (32a)$$

$$\cos\varphi_{21} = \left(1/2\right)\sqrt{4-3\cos^2\varphi_{16}} \quad (32b)$$

$$\cos\varphi_{45} = 1\Big/\sqrt{4-3\cos^2\varphi_{16}} \quad (33a)$$

$$\sin\varphi_{45} = -\sqrt{3}\sin\varphi_{16}\Big/\sqrt{4-3\cos^2{}_{16}} \quad (33b)$$

$$\cos\varphi_{34} = 1-\left(3/2\right)\cos^2\varphi_{16} \quad \text{...} \quad (34a)$$

$$\sin\varphi_{34} = -\left(\sqrt{3}/2\right)\cos\varphi_{16}\sqrt{4-3\cos^2\varphi_{16}} \quad \text{....................................} \quad (34b)$$

$$\cos\varphi_{32} = \left(3\cos^2\varphi_{16}-2\right)/\left(4-3\cos^2\varphi_{16}\right) \quad \text{...................................} \quad (35a)$$

$$\sin\varphi_{32} = -\left(2\sqrt{3}\right)\sin\varphi_{16}/\left(4-3\cos^2\varphi_{16}\right) \quad \text{...................................} \quad (35b)$$

$$\cos_{56} = \cos\varphi_{16}/\sqrt{4-3\cos^2\varphi_{16}} \quad \text{..} \quad (36a)$$

$$\sin\varphi_{56} = -2\sin\varphi_{16}/\sqrt{4-3\cos^2\varphi_{16}} \quad \text{.......................................} \quad (36b)$$

With the help of above equations change in positions of other links with respect to change in position of input link (1) can be determined and is tabulated below:

Table 2: Positions Of Links With Respect To Change In Position Of Input Link 1

Angle in Degrees					
φ_{16}	0	90	180	270	360
φ_{21}	60	0	-60	0	60
φ_{34}	-120	0	120	0	-120
φ_{32}	0	-120	0	120	0
φ_{45}	0	-60	0	60	0
φ_{56}	0	-90	-180	-270	-360

The results tabulated above are represented graphically in figures 5 to 7.

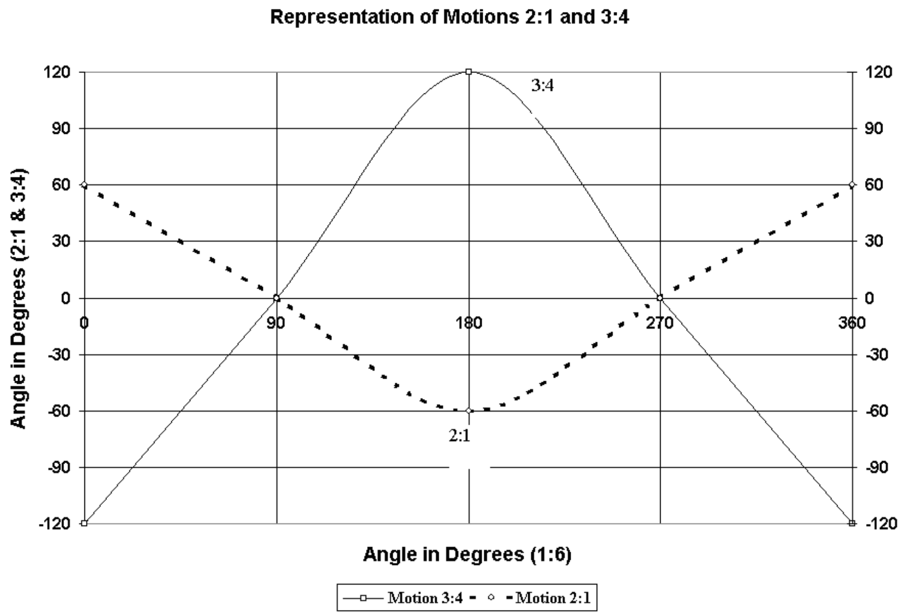

Figure 5

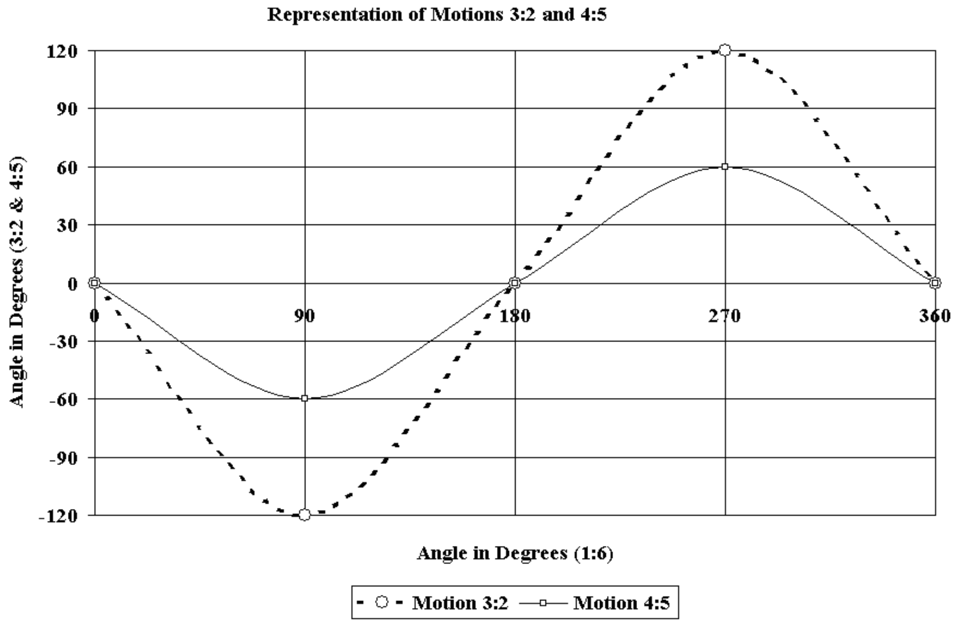

Figure 6

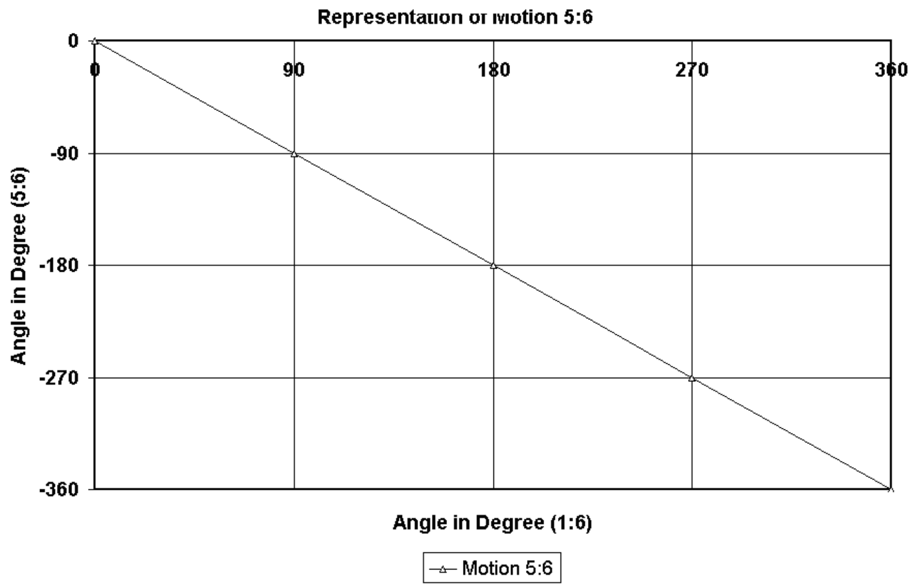

Figure 7

CONCLUSION:

A mixing machine described here employs a very interesting mechanism. Attempt is made here to describe general kinematics of the six link mechanism involving only revolutes, using matrix method. As discussed here that a spatial mechanism containing only revolutes is derived from the seven link kinematics chain. However mechanism described here has only six links including the fixed base. If the link lengths are selected arbitrary, then the mechanism will result in immobility with no degrees of freedom. Mobility of the mechanism is because of specific linear dimensions only. The mobility of the mechanism is proved with this kinematics description. In case of immobility of the mechanism, equations (20) to (31) would have produced contradictory results. Additionally, mobility can also be proved with the help of CAD software & also by actually making the working model of the machine.

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