



EXPLORING PRIME NUMBER DISTRIBUTION PATTERNS: A NUMERICAL BASED STUDY

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ABSTRACT

Prime numbers and their subordinates are exceptionally useful in applied research fields including designing control, criticism, and encryption. Since the instructor's knowledge influences the students' knowledge, showing number juggling requires not simply the educator to be knowledgeable about the subject yet also the instructor to have mathematical knowledge that is helpful for the training and figuring out there. If a teacher wants to be properly understood by their students, they should provide adequate mathematical explanations. We looked into how aspiring math teachers described the idea of a prime number and the methods they used to convey it in the study discussed here. We looked into how aspiring math teachers described the idea of a prime number and the methods they used to convey it in the study discussed here. The study was a qualitative research descriptive survey. The study included 46 individuals who had all finished the abstract algebra classes where the idea in question was taught. A form with three open-ended questions challenging students' understanding of what a prime number is and how to convey it to secondary/high school pupils served as the data collection method. After the information were dissected, the discoveries exhibited that the preservice teachers struggled with characterizing the expression "prime number" and that they needed to apply rules to depict them.

Keywords: *Prime no., odd/even perfect numbers, mathematical study, explanatory strategies.*

1. INTRODUCTION

There is a consensus among most people that mathematics serves as a foundation for all scientific and technical ideas, and that number theory serves as the foundation for mathematics. Finding an integer's unique factor decomposition is a process that is both essential and difficult, as it is strongly dependent on the answers to the issues posed in number theory. When researchers actively participate in their studies, new research methods arise. As a rule, hypothesis can be separated into the accompanying classifications:



rudimentary number hypothesis, which centres around detachability and coinciding; insightful number hypothesis, which is upheld by complex examination; logarithmic number hypothesis because of the development of the study on ring of numbers; mathematical number hypothesis, which depends on the points of view of calculation to make sense of the example of conveyance; and computational number hypothesis, which utilizes PC calculations to take care of specific testing issues. The universe is presently witnessing the rapid application of the notions of number theory in a wide variety of applied domains, including physics, biology, chemistry, communication, acoustics, electronics, encryption, computing, and many others.

A intriguing endeavour in number theory, important to mathematics, and with wide-ranging implications, particularly in cryptography and computer science, is the exploration of prime number distribution patterns through a numerical-based study. This study may be found in the field of number theory. This research goes into the complex realm of prime numbers, beginning with an explanation of their significance and the reasoning behind analysing the patterns of their distribution. An extensive background part provides a precise definition of prime numbers, which is reinforced by historical context and draws attention to major milestones in the study of prime numbers. These key milestones include both ancient techniques like the Sieve of Eratosthenes and more recent discoveries like the Prime Number Theorem. The research then moves on to explain a variety of prime number theorems, with a particular emphasis on the Prime Number Theorem, which describes the asymptotic distribution of primes. This foundation forms the basis for the rest of the study. The research looks deeper into hypotheses, such as the Riemann Hypothesis, and explores well-known distribution patterns, such as clusters of prime numbers and gaps between prime numbers. Additionally, unique primes, such as twin primes and cousin primes, are investigated as part of this project. This study analyses the numerical tools and algorithms that are often used in prime number research. These numerical tools and algorithms include sieves, probabilistic primality tests, and prime-generating algorithms. The investigation of these numerical tools and algorithms helps to facilitate a deeper knowledge of the distribution of prime numbers. Statistical analytic methods such as Benford's Law and the distribution of last digits are investigated in order to unearth possible distribution patterns. In addition, a wide range of visualizations, from histograms to number spirals, are utilized in



order to communicate the elegance and intricacy of the prime number distribution. The nature of this mathematical inquiry, which is one that is ongoing and ever-changing, is underscored by the fact that challenges in the field, existing conjectures, and opportunities for future research are highlighted. The study makes reference to seminal studies and acknowledges the collaborative and interdisciplinary character of research in the distribution of prime numbers. This highlights the ever-expanding frontiers in our understanding of these mysterious and important numbers.

1.1. Prime numbers

On the off chance that $N, n | p, n = 1, W, n = p$, the positive whole number p is supposed to be prime. A composite number is any non-prime number. As indicated by the Central Hypothesis of Math (FTA), each sure number more noteworthy than one might be composed as an interesting result of primes, no matter what the request for the primes. Public and private keys in cryptography are typically calculated using prime numbers. Its efficacy is highly reliant on how challenging it is to factor out huge integers. For his key exchange, for instance, Diffie-Hellman relied on prime numbers. He did this by using a big prime number p as a common modulus for two individuals A_1 and A_2 to exchange secure messages using their individual, secret keys. If two parties A_1 and A_2 each have a private key, a_1 , and agree to share a public key, b , which is less than p , then message (m) from party A_1 to party B_1 can be expressed as: $m^{a_1} b^{a_1} \pmod{p}$ and $m^{a_2} b^{a_2} \pmod{p} = m^{a_2 \cdot 1} \pmod{p} = b^{a_1}$. The secrecy of this exchange is determined by how challenging it is to deduce the shared message from just a_1 and a_2 .

1.2. Finding of prime number distribution patterns

Since the dawn of mathematics, researchers have been interested in studying the patterns of distribution of prime numbers. In spite of the fact that prime numbers appear to be dispersed in a random fashion, certain fascinating patterns and features have nevertheless evolved. For example, Hadamard and de la Vallée-Poussin described the asymptotic distribution of prime numbers in the late 19th century when they developed the Prime Number Theorem. This theorem says that as numbers increase larger, the density of primes drops because there is less room for primes in the larger number. In addition, certain patterns, such as the twin prime conjecture (consecutive primes differing by 2) and the



Goldbach conjecture (expressing even numbers as the sum of two primes), hint at fascinating regularities within the prime number landscape; however, these conjectures have not been demonstrated to be true. These distribution patterns have been investigated using sophisticated mathematical techniques, such as sieve methods and probabilistic models, in order to get a greater understanding of the mysterious qualities of prime numbers.

2. LITERATURE REVIEW

Boujnouni, M. E. (2021). The fact that prime numbers are the "building blocks" of mathematics is one of the primary reasons why they are so useful. Another reason is that there are an infinite number of primes, and the distribution of these numbers does not appear to follow any kind of pattern. In light of these qualities, they were put to use in the construction of effective one-way functions that are utilized in public key cryptography. An effective supervised learning model known as Support Vector Domain Description was utilized as an explorer for the aim of accomplishing this goal.

Boujnouni, M. E. (2021). The PN are understood to be traits that distinguish the outputs of a CS from one another. The Canberra, Euclidean, Jaccard, and Lorentzian distances, in addition to a multidimensional scaling (MDS) algorithm, are used to determine the degree of dissimilarity between the objects in question. These distances are the four metrics that are utilized in the assessment of the differences that exist between the objects. The MDS generates loci that are arranged in accordance with the properties of the objects, and these loci are analysed in the light of the patterns that are emerging. In addition, these patterns are investigated within the Fourier domain from the perspective of fractional calculus.

Segarra, E. (2006). Over the past century and a half, mathematicians have been unable to find a solution to the Riemann zeta hypothesis, which is recognized as one of the 7 Millennium Problems. To put it another way, Riemann hypothesized that the real parts of all of the nontrivial zeroes in his zeta function were equal to the fraction $1/2$. Along the way, we will establish the mathematics required to handle this theory in the goal that by the time we reach the conclusion, the reader will have immersed themselves sufficiently to want to conduct their own inquiry and research into this interesting topic.

Overmars, A., & Venkatraman, S. (2021). In order to investigate Fermat's Christmas theorem, Pythagorean tuples to factor semi-primes have been investigated. If the parity of



the two squares in the equation is opposite one another, the semi-prime can be factorized. To factorize a semi-prime number, you need to discover exactly one sum of three squares that factors into that number. This is what the problem of semi-prime factorization comes down to. In order to achieve four sums of three squares, we make a modification to the Lebesgue identity, which was previously defined as the sum of four squares. From that point forward, these are changed as four different Pythagorean quadruples. In view of the Brahmagupta-Fibonacci character, these four Pythagorean quadruples can be improved on down to two Pythagorean triples. The components that make up the semi-prime are the best normal divisors of the sides that are held inside it.

Mahato, P., & Shah, A. It's possible to think of primes as the "basic building blocks," or even the atoms, of the natural numbers. They are important contributors to the field of number theory. Furthermore, in the modern era of computers and digitalization, prime numbers are of the utmost importance for both computer programmers and scientists in order to solve important real-life issues. The pattern of prime numbers has been the subject of a significant amount of investigation and study for a very long time. A pattern that involves squaring prime numbers is shown in this article. In addition to that, the file contains fifteen distinct types of prime numbers along with the Python code necessary to generate them. Numbers that cannot be factorized into primes are referred to as primes. Primes play a significant role in cryptographic encryption systems since prime factorization is required. In light of this, the study presents prime factorization of composite numbers using the Sieve of Eratosthenes algorithm on several platforms, as well as a time analysis based on the results of this factorization. In addition, a comparison of prime factorization of composite numbers with the amount of time it takes has been plotted on a graph, and a factorization analysis of primes has been proven to be factorized by five distinct algorithms. Additionally presented are two of the most important applications of primes.

Yusuf, B. K., & Mahmood, K. A. B. (2020). A way to deal with grouping was formulated, what separates a prime into two sections: its last digit (alluded to as the "trailer" and comprising of the decreased arrangement of buildups "1, 3, 7 and 9") and different digits (alluded to as the "lead"), the worth of which is expanded by one or the other 1, 2, or 3, so framing a modulo-3 number juggling condition. To investigate a point that was both genuine and testing to settle computationally, the methodology observed both Polignac's and



changed Goldbach's coefficients. A legitimacy test was performed utilizing Sloane's A006988's powers of 10 primes, and precisely 20,064,735,430 lower primes of digits 2 to 12 were found. In numerous direct relapse examination, embracing all things considered cubic terms of indicators was finished (as the following coherent step of Euler's quadratic recipe for primes), and the resultant results were dissected to help with making the Akaike Data Rule (AIC) ideal model with forward determination strategy. A particle is a table with a length of 4,493,869 whole number successions, while an information base has 30 social tables with offices for additional going back over. The essential errand was separated into nuclear units of comparable cases and types. A node that allows parallel processing, investigates 4,044,482,100 successive integers, and stores 30 databases that are continuous with one another. Lower regular numbers were researched by 127 speculative nodes, which brought about the creation of primes that were then saved in 114,300 tables that were scattered all through 3,810 data sets. Among the noteworthy assets was a Veriton S6630G PC framework with 3.60GHz Intel(R) Center (TM) i7-4790 computer processor handling and 7.86GB of useable Slam.

Farshi, E. which postulates that if a straightforward function is applied in an iterative fashion to any starting odd number, the result will finally be the number 1. The results of our research offer fresh perspectives on the part that topological characteristics and prime factors play in determining behaviour. After analysing the topological features of the subspace H_3 and defining it as consisting of numbers that are divisible by 3, we discover that the basic group of this subspace is not trivial. We observe an intersection between the sequences for odd numbers and the numbers in H_3 , which suggests that the sequences for odd numbers may eventually reach a number in H_3 , which indicates that additional inquiry is warranted. When we compare the characteristics of sequences that intersect with H_3 to those that do not, we discover that there are significant changes in the length of the sequence, the maximum value, and the convergence rate. In addition to this, we determine the distribution of primes inside each sequence by generating prime factorizations for each number in the sequences and analysing the results. Our research demonstrates that there are relationships between the number of prime factors, the size of the greatest prime factor, and sequence features such as maximum value and average length. The findings throw open the possibility of new lines of inquiry, such as investigating number-theoretic properties,



algebraic properties, asymptotic analysis, or linkages to other topics in number theory or computer science.

3. RESEARCH METHODOLOGY

3.1. Research Model

This research was a personal examination study, and a making sense of diagram model was utilized as the technique considering how our objective was to pick the idea clarifications that PMTs have about prime numbers and their different explanatory strategies.

3.2. Members

The people were picked PMTs who had gotten done with the speculative polynomial mathematical courses and had gotten the as of late referred to knowledge while sought after such courses. The members remembered 46 fourth-year PMTs for the Branch of Science Instructing (the PMTs were taught in a similar class) and alumni of the Division of Math who were going through educator's preparation (the PMTs were instructed in a similar class of the educational development preparing). The understudies who sign up for the Division of Science Showing will proceed to become math teachers in optional schools, while the understudies who sign up for the Branch of Math will proceed to become math teachers in secondary schools.

3.3. Data collection

We employed a questionnaire with three open-ended questions to conduct our research. The requests were as per the following: 1) "What definitively is a PN?" 2) "Are there a few other expected definitions for PNs that aren't recorded here? Assuming such is the situation, might you at any point assist with making sense of these definitions? When developing the questionnaire for the survey, we took into consideration the feedback of two members of the faculty, one of whom was responsible for teaching the abstract algebra class. The participants were provided with a printed copy of the open-ended questions, and they were given fifty minutes to respond to the questions.

4. DATA ANALYSIS

During the time spent examining the information, one of the scientists at first played out the examination; then, at that point, to ensure the precision of the encoding, the information



was reanalysed by an alternate specialist at a later moment. It was discovered that 95% of the encoding was consistent. The contentious codes were dissected in great detail, and the researchers came to a decision that everyone could agree with.

Table 1 The precision of the PN definitions in their entirety

<i>Exactness of the definition</i>		<i>Measures</i>	<i>Recurrence</i>	
			<i>first question</i>	<i>2nd question</i>
<i>Exactness</i>	<i>Fitting/ improper</i>	<i>Fundamental and sufficient</i>	9	1
		<i>Fundamental however in sufficient</i>	50	25
		<i>Mostly fundamental yet lacking</i>	1	10
		<i>Neither fundamental nor satisfactory</i>	1	10
		<i>Void</i>	1	10

Interpretation According to the disclosures that are presented in Table 1, the precision conditions of the PN definitions that were given by the PMTs contained both legitimate and improper definitions. While proper definitions meet both the fundamental and satisfactory standards, unseemly definitions were delegated fundamental however insufficient, halfway fundamental yet lacking, or neither fundamental nor sufficient. While fitting definitions have the fundamental and sufficient models, unseemly definitions were arranged as fundamental yet lacking. On the other hand, it was found that the definitions for PN may be broken down into two categories: essential and adequate, and essential but insufficient. The vast majority of PMTs provided definitions of PN, all of which fell into the category of being necessary but insufficient (50 replies fell into this category).



Table 2 prime numbers definition

<i>Prime numbers</i>	<i>Fundamental and satisfactory</i>	<i>Fundamental however insufficient</i>	<i>Incompletely fundamental yet lacking</i>	<i>Neither fundamental nor sufficient</i>
<i>Number</i>		<p>b) which has no denominator other than 1 and itself ($n = 31$)</p> <p>c) which cannot be separated by any number other than 1 and itself under the condition $a \neq 0$ ($n = 1$)</p>		
<i>Natural number</i>	a) equivalent to or more prominent than 2, which has no certain denominators other than 1 and itself ($n = 3$)	<p>d) regular numbers that must be separated by 1 and itself ($n = 2$)</p> <p>e) regular numbers, aside from 1, that must be separated by 1 and itself ($n = 1$)</p> <p>f) regular numbers that have just two positive denominators ($n = 1$)</p> <p>g) regular numbers equivalent to or more prominent than 2, that must be separated by 1 and itself ($n = 1$)</p>		
<i>Positive integer</i>		<p>h) positive whole numbers more prominent than 1, that have no denominators other than 1 and itself ($n = 2$)</p>		
<i>Integer</i>		<p>i) positive whole numbers that have no denominators other than 1 and itself ($n = 2$) Whole number</p> <p>j) numbers that have no denominators other than 1 and itself ($n = 2$)</p>		



Interpretation This table provides a categorization of the many kinds of numbers, with a special emphasis on prime numbers, natural numbers, positive integers, and integers, according to whether or not they are regarded necessary or sufficient for a variety of criteria. Because of the way that they fulfil the rules of having no certain denominators other than 1 and themselves, prime numbers ($n = 3$) are alluded to as being vital and adequate. Natural numbers, which have a value of 31, are considered necessary but insufficient since they do not possess the quality of being divided by only one and by themselves. Positive integers, with a n value of 2, belong to the class of integers that do not have any denominators other than themselves and the number 1, which makes them necessary but not sufficient. This table investigates the qualities and classifications of these numbers according to the characteristics of their divisibility, and it establishes the relevance of prime numbers in relation to the other numbers in this set.

5. CONCLUSION

When attempting to define the idea of PN, we discovered that PMTs produced explanations that were both accurate and wrong. It was discovered that the vast majority of definitions are not accurate. Just three of the PMTs featured that the positive denominators of PNs were 1 and the veritable number, and they utilized the definition that can be found in the Wolfram Math World word reference. As a result, they provided an explanation that was true. However, with the exception of these answers, all of the others offered explanations that were either inadequate or unrelated, and as a result, they were categorized as inaccurate explanations. This showed that the PMTs were truly unsure about this subject. The going with bungles occurred considering their reasoning: the meaning of PNs in more noteworthy definition sets, for example, "number," "number," or "normal number," without feeling that they were extraordinary whole numbers; acknowledging zero, one, and doubtful whole numbers as PNs; neglecting to acknowledge the presence of basic denominators among the denominators of the number; endeavouring to make sense of relative PNs or which numbers would be PNs under the state of one PN. In the research carried out by Cavey et al. (2015), students were asked to describe PNs and rate their preferences on usability and openness. The researchers found that students made errors



that were similar to one another. Also, they found that most of the definitions given by PMTs had critical expressions, for example, "... which doesn't have denominators."

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