AN ALGORITHM FOR SOLVING A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM

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Abstract: In this paper, a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. An algorithm to find the efficient cost-time trade off pairs in a capacitated fixed charge bi-criterion indefinite quadratic transportation problem is developed. The algorithm is based on the concept of solving the indefinite quadratic fixed charge transportation problem and reading the corresponding time from the time matrix. It is illustrated with the help of a numerical example.

Keywords: optimum time cost trade off, capacitated transportation problem, fixed charge, bi-criterion indefinite quadratic transportation problem.

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1. INTRODUCTION:

In the classical transportation problem, the cost of transportation is directly proportional to the number of units of the commodity transported. But in real world situations, when a commodity is transported, a fixed cost is incurred in the objective function. The fixed cost may represent the cost of renting a vehicle, landing fees at an airport, set up cost for machines etc. The fixed charge transportation problem was originally formulated by G.B Dantzig and W. Hirisch [13] in 1954. After that, several procedures for solving fixed charge transportation problems were developed.

Sometimes, there may exist emergency situations such as fire services, ambulance services, police services etc when the time of transportation is more important than cost of transportation. Hammer[12], Szwarc[14], Garfinkel et al. [7], Bhatia et al. [5] and many others have studied the time minimizing transportation problem which is a special case of bottleneck linear programming problems.

Another important class of transportation problem consists of capacitated transportation problem. Many researchers like Gupta et.al. [8-11], Dahiya et.al. [6] have contributed in this field. In 1976, Bhatia et.al. [4] provided the time cost trade off pairs in a linear transportation problem. Then in 1994, Basu et.al. [3] developed an algorithm for finding the optimum time cost trade off pairs in a fixed charge linear transportation problem giving same priority to cost and time.

Another class of transportation problems, where the objective function to be optimized is the product of two linear functions which gives more insight into the situation than the optimization of each criterion. Arora et. al. [1-2] have contributed in the field of indefinite quadratic transportation problem.

In this paper, a capacitated fixed charge indefinite quadratic transportation problem giving same priority to cost and time is studied. An algorithm to identify the efficient cost time trade off pairs for the problem is developed.
2. MATHEMATICAL MODEL FOR A CAPACITATED FIXED CHARGE BI-CRITERION INDEFINITE QUADRATIC TRANSPORTATION PROBLEM:

(P1): \[ \min \left\{ \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \left( \sum_{i \in I} \sum_{j \in J} d_{ij} x_{ij} \right) + \sum_{i \in I} \max t_{ij} / x_{ij} > 0 \right\} \]

subject to

\[ \sum_{j \in J} x_{ij} \leq a_i; \forall i \in I \]  
(1)

\[ \sum_{i \in I} x_{ij} = b_j; \forall j \in J \]  
(2)

\[ l_{ij} \leq x_{ij} \leq u_{ij}; \forall (i, j) \in I \times J \]  
(3)

I = \{1, 2, ... m\} is the index set of m origins.

J = \{1, 2, ..., n\} is the index set of n destinations.

\[ x_{ij} \] = number of units transported from origin i to the destination j.

\[ c_{ij} \] = variable cost of transporting one unit of commodity from \( i^{th}\) origin to the \( j^{th}\) destination.

\[ d_{ij} \] = the per unit damage cost or depreciation cost of commodity transported from \( i^{th}\) origin to the \( j^{th}\) destination.

\[ l_{ij} \text{ and } u_{ij} \] are the bounds on number of units to be transported from \( i^{th}\) origin to \( j^{th}\) destination.

\[ t_{ij} \] is the time of transporting goods from \( i^{th}\) origin to the \( j^{th}\) destination.

\( F_i \) is the fixed cost associated with \( i^{th}\) origin. The fixed cost \( F_i \) depends upon the amount supplied from the \( i^{th}\) origin to different destinations and is defined as follows.

\[ F_i = \sum_{l=1}^{p} F_{il} \delta_{il}, \quad i=1, 2, 3....m, \quad l=1,2,3...p \]

where \( \delta_{il} = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} x_{ij} > a_{il} \\ 0 & \text{otherwise} \end{cases} \) for \( l=1,2,3......p, \ i=1,2,......m \)

Here, 0 = \( a_{i1} < a_{i2} \ ... < a_{ip} \). Also \( a_{i1} \ , \ a_{i2} \ ... \ , \ a_{ip} \ (i = 1,2, ... m) \) are constants and \( F_{il} \) are the fixed costs \( \forall i= 1, 2 ...m, \ l= 1,2 ...p \)
In the problem (P1), we need to minimize the total transportation cost and depreciation cost simultaneously to be transported from the i\textsuperscript{th} origin to the j\textsuperscript{th} destination. Also, we need to find the different cost-time tradeoff pairs.

3. THEORETICAL DEVELOPMENT:

Since $\sum_{i} a_{i} > \sum_{j} b_{j}$, the given problem is not a balanced problem. Therefore, we introduce a dummy destination (n+1) with demand $b_{n+1} = \sum_{i} a_{i} - \sum_{j} b_{j}$. The cost and time allocated in this (n+1)\textsuperscript{th} column is zero. The resulting balanced problem is given below.

\[
(P1): \min \left\{ \left( \sum_{i \in I, j \in J'} c_{ij} x_{ij} \right) \left( \sum_{i \in I, j \in J'} d_{ij} x_{ij} \right) + \sum_{i \in I} F_{i}, \max \left( t_{ij} / x_{ij} > 0 \right) \right\}
\]

subject to

\[
\sum_{j=1}^{n+1} x_{ij} = a_{i}; \forall i \in I
\]

\[
\sum_{i=1}^{m} x_{ij} = b_{j}; \forall j \in J'
\]

\[
l_{ij} \leq x_{ij} \leq u_{ij}; \forall i \in I, j \in J
\]

where $J' = \{1, 2, 3, \ldots, n, n+1\}$ and $x_{i, n+1} \geq 0; l_{i, n+1} = 0; u_{i, n+1} = 0; \forall i \in I$

$c_{i, n+1} = 0 = d_{i, n+1} = t_{i,n+1}; \forall i \in I$

$b_{n+1} = \sum_{i=1}^{m} a_{i} - \sum_{j=1}^{n} b_{j}$

$F_{i}$ for $i=1, 2, \ldots, m$ are defined as in problem (P1).

In order to solve the problem (P1'), we separate it into two problems (P2) and (P3) where

(P2): minimize the cost function $\left\{ \left( \sum_{i \in I, j \in J'} c_{ij} x_{ij} \right) \left( \sum_{i \in I, j \in J'} d_{ij} x_{ij} \right) + \sum_{i \in I} F_{i} \right\}$ subject to (4), (5) and (6).

(P3): minimize the time function $\max \left( t_{ij} / x_{ij} > 0 \right)$ subject to (4), (5) and (6).

To obtain the set of efficient cost-time tradeoff pairs, we first solve the problem (P2) and read the time with respect to the minimum cost $Z$ where time $T$ is given by the problem (P3).
At the first iteration, let $Z^*_1$ be the minimum total cost of the problem (P2). Find all alternate solutions i.e., solutions having the same value of $Z = Z^*_1$. Let these solutions be $X_1, X_2, \ldots, X_n$. Corresponding to these solutions, find the time $T^*_1 = \min_{X_1, X_2, \ldots, X_n} \left\{ \max_{i \in I, j \in J} \left( t_{ij} / x_{ij} > 0 \right) \right\}$. Then $(Z^*_1, T^*_1)$ is called the first cost time trade off pair. Modify the cost with respect to the time so obtained i.e., define $c_{ij} = \begin{cases} M & \text{if } t_{ij} \geq T^* \\ c_{ij} & \text{if } t_{ij} < T^* \end{cases}$ and form the new problem (P2') and find its optimal solution and all feasible alternate solutions. Let the new value of $Z$ be $Z^*_2$ and the corresponding time is $T^*_2$, then $(Z^*_2, T^*_2)$ is the second cost time trade off pair. Repeat this process. Suppose that after $q$th iteration, the problem becomes infeasible. Thus, we get the following complete set of cost-time trade off pairs. $(Z^*_1, T^*_1), (Z^*_2, T^*_2), \ldots, (Z^*_q, T^*_q)$, where $Z^*_1 \leq Z^*_2 \leq Z^*_3 \leq \ldots \leq Z^*_q$ and $T^*_1 > T^*_2 > T^*_3 \ldots \ldots > T^*_q$. The pairs so obtained are pareto optimal solution of the given problem. Then we identify the minimum cost $Z^*_1$ and minimum time $T^*_q$ among the above trade off pairs. The pair $(Z^*_1, T^*_q)$ with minimum cost and minimum time is termed as the ideal pair which can not be achieved in practical situations.

**Theorem 1:** Let $X = \{X_{ij}\}$ be a basic feasible solution of problem (P2) with basis matrix $B$. Then it will be an optimal basic feasible solution if

\[
R_y^1 = \theta_{ij} \left[ z_1(d_{ij} - z^1_{ij}) + z_2(c_{ij} - z^2_{ij}) + \theta_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) \right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1
\]

and

\[
R_y^2 = \theta_{ij} \left[ \theta_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) - z_1(d_{ij} - z^1_{ij}) - z_2(c_{ij} - z^1_{ij}) \right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2
\]

such that

\[
\begin{align*}
u^1_i + v^1_j &= c_{ij} & \forall (i, j) \in B \\
u^2_i + v^2_j &= d_{ij} & \forall (i, j) \in B \\
u^1_i + v^1_j &= z^1_{ij} & \forall (i, j) \in N_1 \text{ and } N_2 \\
u^2_i + v^2_j &= z^2_{ij} & \forall (i, j) \in N_1 \text{ and } N_2
\end{align*}
\]
ΔF_{ij} is the change in fixed cost $\sum_{i \in I} F_i$ when some non basic variable $x_{ij}$ undergoes change by an amount of $\theta_{ij}$.

$Z_1 =$ value of $\sum_{i \in I} \sum_{j \in J} c_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis $B$.

$Z_2 =$ value of $\sum_{i \in I} \sum_{j \in J} d_{ij}x_{ij}$ at the current basic feasible solution corresponding to the basis $B$.

$\theta_{ij} =$ level at which a non basic cell $(i,j)$ enters the basis replacing some basic cell of $B$.

$N_1$ and $N_2$ denotes the set of non basic cells $(i,j)$ which are at their lower bounds and upper bounds respectively.

**Note:** $u^1_{ij}$, $v^1_{ij}$, $u^2_{ij}$, $v^2_{ij}$ are the dual variables which are determined by using equations (7) to (10) and taking one of the $u^i$'s or $v^i$'s as zero.

**Proof:** Let $Z^0$ be the objective function value of the problem (P2).

Let $Z^0 = Z_1Z_2 + F^0$ where $F^0 = \sum_{i \in I} F_i$.

Let $\hat{z}$ be the objective function value at the current basic feasible solution $\hat{X} = \{x_{ij}\}$ corresponding to the basis $B$ obtained on entering the non basic cell $x_{ij} \in N_1$ into the basis which undergoes change by an amount $\theta_{ij}$ and is given by min$(u_{ij} - l_{ij}; x_{ij} - l_{ij}$ for all basic cells $(i,j)$ with a (- $\theta$) entry in the $\theta$-loop; $u_{ij} - x_{ij}$ for all basic cells $(i,j)$ with a (+ $\theta$) entry in the $\theta$-loop).

Then $\hat{z} = \left[ z_1 + \theta_{ij}(c_{ij} - z^1_{ij}) \right] \left[ z_2 + \theta_{ij}(d_{ij} - z^2_{ij}) \right] + F^0 + \Delta F_{ij}$

$\hat{z} - Z^0 = \left[ z_1z_2 + \theta_{ij}z_1(d_{ij} - z^2_{ij}) + z_2\theta_{ij}(c_{ij} - z^1_{ij}) + \theta^2_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) - z_1z_2 \right] + \Delta F_{ij}$

$\hat{z} - Z^0 = \theta_{ij} \left[ z_1(d_{ij} - z^2_{ij}) + z_2(c_{ij} - z^1_{ij}) + \theta_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) \right] + \Delta F_{ij}$

This basic feasible solution will give an improved value of $z$ if $\hat{z} < Z^0$. It means if $\theta_{ij} \left[ z_1(d_{ij} - z^2_{ij}) + z_2(c_{ij} - z^1_{ij}) + \theta_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) \right] + \Delta F_{ij} < 0$ \hspace{1cm} (11)

Therefore one can move from one basic feasible solution to another basic feasible solution on entering the cell $(i,j) \in N_1$ into the basis for which condition (11) is satisfied.

It will be an optimal basic feasible solution if

$R^1_{ij} = \theta_{ij} \left[ z_1(d_{ij} - z^2_{ij}) + z_2(c_{ij} - z^1_{ij}) + \theta_{ij}(c_{ij} - z^1_{ij})(d_{ij} - z^2_{ij}) \right] + \Delta F_{ij} \geq 0; \forall (i,j) \in N_1$
Similarly, when non basic variable \( x_{ij} \in N_2 \) undergoes change by an amount \( \theta_{ij} \) then

\[
\hat{z} - z^0 = \theta_{ij} \left[ \theta_{ij}(c_{ij}-z_{ij}^1) + (d_{ij}-z_{ij}^2) - z_1(d_{ij}-z_{ij}^2) - z_2(c_{ij}-z_{ij}^1) \right] + \Delta F_{ij} < 0
\]

It will be an optimal basic feasible solution if

\[
R_{ij}^2 = \theta_{ij} \left[ \theta_{ij}(c_{ij}-z_{ij}^1) + (d_{ij}-z_{ij}^2) - z_1(d_{ij}-z_{ij}^2) - z_2(c_{ij}-z_{ij}^1) \right] + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2
\]

4. ALGORITHM:

**Step 1:** Given a capacitated fixed charge bi-criterion indefinite quadratic transportation problem (P1). Introduce a dummy destination to form the related balanced transportation problem (P1’) and then separate it in to two problems (P2) and (P3). Find a basic feasible solution of problem (P2) with respect to variable cost only. Let B be its corresponding basis.

**Step 2:** Calculate the fixed cost of the current basic feasible solution and denote it by \( F(\text{current}) \)

where \( F(\text{current}) = \sum_{i=1}^{m} F_i \)

**Step 3(a):** Find \( \Delta F_{ij} = F(\text{NB}) - F(\text{current}) \) where \( F(\text{NB}) \) is the total fixed cost obtained when some non basic cell \((i,j)\) undergoes change.

**Step 3(b):** Calculate \( \theta_{ij}, (c_{ij}-z_{ij}^1), (d_{ij}-z_{ij}^2), z_1, z_2 \) for all non basic cells such that

\[
\begin{align*}
  u_{i}^{1} + v_{j}^{1} &= c_{ij} & \forall (i, j) &\in B \\
u_{i}^{2} + v_{j}^{2} &= d_{ij} & \forall (i, j) &\in B \\
u_{i}^{1} + v_{j}^{1} &= z_{ij}^{1} & \forall (i, j) &\in N_1 \text{ and } N_2 \\
u_{i}^{2} + v_{j}^{2} &= z_{ij}^{2} & \forall (i, j) &\in N_1 \text{ and } N_2 \\
\end{align*}
\]

\( Z_1 = \) value of \( \sum_{i\epsilon I} \sum_{j\epsilon J} c_{ij}x_{ij} \) at the current basic feasible solution corresponding to the basis B.

\( Z_2 = \) value of \( \sum_{i\epsilon I} \sum_{j\epsilon J} d_{ij}x_{ij} \) at the current basic feasible solution corresponding to the basis B.

\( \theta_{ij} = \) level at which a non basic cell \((i,j)\) enters the basis replacing some basic cell of B.

\( N_1 \) and \( N_2 \) denotes the set of non basic cells \((i,j)\) which are at their lower bounds and upper bounds respectively.

**Note:** \( u_{i}^{1}, v_{j}^{1}, u_{i}^{2}, v_{j}^{2} \) are the dual variables which are determined by using above equations and taking one of the \( u_{i}^{5} \) or \( v_{j}^{5} \) as zero.
Step 3(c): Find $R_{ij}^1 \forall (i, j) \in N_1$ and $R_{ij}^2 \forall (i, j) \in N_2$ where

$$R_{ij}^1 = 0_{ij} + z_2(c_{ij} - z_{ij}^1) + 0_{ij}(d_{ij} - z_{ij}^2) + \Delta F_{ij} \geq 0; \forall (i, j) \in N_1$$

and

$$R_{ij}^2 = 0_{ij}(c_{ij} - z_{ij}^1)(d_{ij} - z_{ij}^2) - z_2(c_{ij} - z_{ij}^1) - z_2(c_{ij} - z_{ij}^2) + \Delta F_{ij} \geq 0; \forall (i, j) \in N_2$$

Step 4: If $R_{ij}^1 \geq 0 \forall (i, j) \in N_1$ and $R_{ij}^2 \geq 0 \forall (i, j) \in N_2$ then the current solution so obtained is the optimal solution to (P2). Go to step 5. Otherwise, some $(i,j) \in N_1$ for which $R_{ij}^1 < 0$ or some $(i,j) \in N_2$ for which $R_{ij}^2 < 0$ will undergo change. Go to step 2.

Step 5: Let $Z^1$ be the optimal cost of $(P2')$ yielded by the basic feasible solution $\{y'_{ij}\}$. Find all alternate solutions to the problem (P2) with the same value of the objective function. Let these solutions be $X_1, X_2, ..., X_n$ and $T_1 = \min_{X_1, X_2, ..., X_n} \left\{ \max_{i \in I, j \in J} \left( t_{ij} / x_{ij} > 0 \right) \right\}$. Then the corresponding pair $(Z^1, T^1)$ will be the first time cost trade off pair for the problem (P1). To find the second cost-time trade off pair, go to step 6.

Step 6: Define $c_{ij}^1 = \begin{cases} M & \text{if } t_{ij} \geq T^1 \\ c_{ij} & \text{if } t_{ij} < T^1 \end{cases}$

where $M$ is a sufficiently large positive number. Form the corresponding capacitated fixed charge quadratic transportation problem with variable cost $c_{ij}^1$. Repeat the above process till the problem becomes infeasible. The complete set of time cost trade off pairs of (P1) at the end of $q^{th}$ iteration are given by $(Z^1, T^1), (Z^2, T^2), ..., (Z^q, T^q)$ where $Z^1 \leq Z^2 \leq ... \leq Z^q$ and $T^1 > T^2 > ... > T^q$.

Remark 2: The pair $(Z^1, T^q)$ with minimum cost and minimum time is the ideal pair which can not be achieved in practice except in some trivial case.

Convergence of the algorithm: The algorithm will converge after a finite number of steps because we are moving from one extreme point to another extreme point and the problem becomes infeasible after a finite number of steps.

5. NUMERICAL ILLUSTRATION:

Consider a 3 x 3 capacitated fixed charge bi-criterion indefinite quadratic transportation problem. Table 1 gives the values of $c_{ij}, d_{ij}, a_i, b_j$ for $i=1,2,3$ and $j=1,2,3$.

Table 1: Cost matrix of problem (P1)
Note: values in the upper left corners are $c_{ij}$,s and values in lower left corners are $d_{ij}$,s for $i=1,2,3$ and $j=1,2,3$. 

$1 \leq x_{11} \leq 10, 2 \leq x_{12} \leq 10, 0 \leq x_{13} \leq 5, 0 \leq x_{21} \leq 15, 3 \leq x_{22} \leq 15, 1 \leq x_{23} \leq 20, 0 \leq x_{31} \leq 20, 0 \leq x_{32} \leq 13, 0 \leq x_{33} \leq 25$

$F_{11} = 100, F_{12} = 50, F_{13} = 50, F_{21} = 150, F_{22} = 100, F_{23} = 50, F_{31} = 200, F_{32} = 150, F_{33} = 100$

$F_i = \sum_{j=1}^{3} F_{ij} \delta_{il}$ for $l=1,2,3$ where for $i=1,2,3$

$\delta_{i1} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$

$\delta_{i2} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 10 \\ 0 & \text{otherwise} \end{cases}$

$\delta_{i3} = \begin{cases} 1 & \text{if } \sum_{j=1}^{3} x_{ij} > 20 \\ 0 & \text{otherwise} \end{cases}$

Table 2 gives the values of $t_{ij}$,s for $i=1,2,3$ and $j=1,2,3$

Table 2: values of $t_{ij}$

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₁</td>
<td>15</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>O₂</td>
<td>10</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>
Introduce a dummy destination in Table 1 with \( c_{ij} = 0 = d_{ij} \) for all \( i = 1, 2, 3 \) and \( b_4 = 70 \)

Now we find an initial basic feasible solution of problem (P2) which is given in Table 3 below.

**Table 3: A basic feasible solution of problem (P2)**

<table>
<thead>
<tr>
<th>i</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>( u^1_i )</th>
<th>( u^2_i )</th>
<th>F(current)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>O2</td>
<td>4</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>250</td>
</tr>
<tr>
<td>O3</td>
<td>2</td>
<td>20</td>
<td>1</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>450</td>
</tr>
</tbody>
</table>

**Note:** entries of the form \( a \) and \( b \) represent non-basic cells which are at their lower and upper bounds respectively. Entries in bold are basic cells.

\[ F(\text{current}) = 800, \; z_1 = 177, \; z_2 = 347 \]

**Table 4: Calculation of optimality condition**

<table>
<thead>
<tr>
<th>NB</th>
<th>O1D1</th>
<th>O1D2</th>
<th>O1D3</th>
<th>O3D1</th>
<th>O3D2</th>
<th>O3D3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{ij} )</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>( c_{ij} - z^1_{ij} )</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>-2</td>
<td>-5</td>
<td>-1</td>
</tr>
<tr>
<td>( d_{ij} - z^2_{ij} )</td>
<td>1</td>
<td>-5</td>
<td>-3</td>
<td>-1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>( F(\text{NB}) )</td>
<td>750</td>
<td>800</td>
<td>800</td>
<td>850</td>
<td>850</td>
<td>850</td>
</tr>
<tr>
<td>( \Delta F_{ij} )</td>
<td>-50</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>( R^1_{ij} )</td>
<td>4747</td>
<td>252</td>
<td>7256</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2_{ij} )</td>
<td>5348</td>
<td>12860</td>
<td>5255</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since $R_{ij}^1 \geq 0 \forall (i, j) \in N_1$ and $R_{ij}^2 \geq 0; \forall (i, j) \in N_2$, the solution in table 3 is an optimal solution of $(P2)$. Therefore minimum cost $= Z^1 = (177 \times 347) + 800 = 62219$ and corresponding time is $T^1 = 15$. Hence the first time cost trade off pair is $(62219,15)$.

Define $c_{ij}^I = \begin{cases} M & \text{if } t_{ij} \geq T^I = 15 \\ c_{ij} & \text{if } t_{ij} < T^I = 15 \end{cases}$ and solving the given problem, we get the second cost time trade off pair as $(62219,13)$. Similarly, third cost-time trade off pair is $(62471,12)$. After that the problem becomes infeasible. Hence the cost time trade off pairs are $(62219,15),(62219,13),(62471,12)$.

6. CONCLUSION

In order to solve a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, we separate the problem into two problems. One of them being an indefinite quadratic programming problem has its optimal solution at its extreme point. After finding the optimal solution we read the time from the time matrix corresponding to $x_{ij} > 0$. Then we define the new cost and new problem with these costs to find the second cost-time trade off pair. The process is repeated till the problem becomes infeasible. The process must end after a finite number of steps because our algorithm moves from one extreme point to another which is finite in number.

REFERENCES


