## ON THE DERIVATION OF THE STABILITY FUNCTION OF A NEW NUMERICAL SCHEME OF ORDER SEVEN FOR THE SOLUTION OF INITIAL VALUE PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS

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#### Abstract

This paper presents the derivation and the analysis of the stability function of a new numerical scheme. It also compares the derived stability function with some existing ones. Stability is a very desirable property for any numerical integration algorithm, particularly if the initial value problem under consideration is to be stiff or stiff oscillatory.


 Keywords: Numerical Integrator, Ordinary Differential Equation, Stability Function[^0]
### 1.0 INTRODUCTION

Any error introduced at any stage of computation can produce unstable numerical results if the problem under consideration is bad or the solution is not well posed. A numerical solution to a differential equation is unstable if as the procedure for its computation progresses, the numerical solutions of ordinary differential equations deviate significantly from the true solution. The peculiar nature of numerical solution of ordinary differential equation demands that we examine the stability properties of any newly derived schemes. We therefore consider in this paper the derivation of the stability polynomial of a new onestep scheme by [11] with a view to assessing its reliability. There are many excellent and exhaustive texts on this subject that may be consulted, such as [1], [3], [4], [5], [6] and [8]. We shall investigate the stability properties using the well-known A-stability model equation otherwise known as the test equations.

$$
\begin{equation*}
y^{\prime}=\lambda y, y\left(x_{0}\right)=y_{0} \tag{1.0}
\end{equation*}
$$

Suggested by [4], we shall also assume that $\operatorname{Re}(\lambda)<0$ and that the equation has a steady state solution.

$$
\begin{equation*}
y^{\prime}(x)=y\left(x_{0}\right) e^{\lambda x} \tag{1.1}
\end{equation*}
$$

Thus, a desired requirement of any integrator would be that the steady solution $y(x)$ of the test equation agrees, as much as possible with the solution of the associated difference equation which is the new scheme that approximates the differential equation.

Over the years, a large number of methods suitable for solving ordinary equations have been proposed. Generally, the efficiency of any of these methods depends on the stability and accuracy properties. The accuracy properties of different methods are usually compared by considering the order of convergence as well as the truncation error coefficient of the various methods.
[11] proposed a numerical integration of order six which is particularly well suited for the solution of initial value problems having oscillatory or exponential solutions. This method was based on the local representation of the theoretical solution $y(x)$ to the initial value problem of the form

$$
\begin{equation*}
y^{\prime}=f(x, y), y(a)=\eta \tag{1.2}
\end{equation*}
$$

in the interval $\left(x_{t}, x_{t+1}\right)$ by a polynomial interpolating function

$$
\begin{equation*}
F(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4}+b\left(r e a \phi^{\rho x+\mu}\right) \tag{1.3}
\end{equation*}
$$

where $a_{0}, a_{1}, a_{2}, a_{3}$ and $b$ are real undetermined coefficients, $\rho$ and $\mu$ are complex parameters.

### 2.0 THE BASIC INTERPOLANT

Let us assume that the theoretical solution $y(x)$ to the IVP problem
$y^{\prime}=f(x, y), y(a)=\eta$ can be represented locally in the interval $\left(x_{t}, x_{t+1}\right), t \geq 0$ by the polynomial interpolating function.
$F(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4}+b\left(\right.$ reale $\left.^{\rho x+\mu}\right)$
Where $a_{0}, a_{1}, a_{2}, a_{3}, a_{4}$ and $b$ are real undetermined coefficients, $\rho$ and $\mu$ are complex parameters.

If we put

$$
\begin{equation*}
\rho=\rho_{1}+i \rho_{2} \tag{1.5}
\end{equation*}
$$

And $\rho=i \sigma=i^{2}=-1$ in (1.4), we obtain the following interpolating function

$$
\begin{equation*}
F(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4}+b e^{\rho_{1} x} \cos \left(\rho_{2} x+\sigma\right) \tag{1.6}
\end{equation*}
$$

If we move further to define the functions $R(x)$ and $\theta(x)$ as follows

$$
\begin{align*}
& R(x)=e^{\rho_{1} x}  \tag{1.7}\\
& \theta(x)=\rho_{2} x+\sigma
\end{align*}
$$

From (1.6) and (1.7), we obtain

$$
\begin{equation*}
F(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+a_{4} x^{4}+b R(x) \cos (\theta x) \tag{1.8}
\end{equation*}
$$

We shall assume that $y_{t}$ is the numerical solution to the theoretical solution $y\left(x_{t}\right)$ and $f_{t}=f\left(x_{t}, y_{t}\right)$. We define mesh point as follows:
$x_{t}=a+t h$,
$t=0,1,2,3, \ldots$
With some imposed constraints on the interpolating function (1.8) then, we came out with a scheme of order seven,

$$
\begin{aligned}
& y_{t+1}-y_{t}= \\
& a_{1}=\left\{f_{t}-f_{t}{ }^{1}-\left[f_{t}{ }^{2}-\left[f_{t}{ }^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right.\right. \\
& \left.\frac{\left[\left(\rho_{1}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2} \\
& -\frac{1}{2}\left[f_{t}{ }^{2}-\left[f_{t}{ }^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right. \\
& \left.\left[\frac{\left[\left(\rho_{1}{ }^{3}-3 \rho_{1} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2}-f_{t}{ }^{3}\right\} h \\
& -\left\{\frac{1}{6}\left[\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{3}\right. \\
& \left.+\frac{\left[\rho_{1} \cos \theta_{t}-\rho_{2} \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right\} h \\
& +\frac{1}{2}\left\{f_{t}{ }^{1}-\left[f_{t}{ }^{2}-\left[f_{t}{ }^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+\text { th })-\right.\right. \\
& \left.\frac{\left[\left(\rho_{1}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}{ }^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h) \\
& \left.-f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2}- \\
& \left.\left[\frac{\left[\left(\rho_{1}{ }^{2}-\rho_{2}{ }^{2}\right) \cos \theta_{t}-2 \rho_{1} \rho_{2} \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\left(2 a h+(1+2 t) h^{2}\right)\right\} \\
& a_{3}=\frac{1}{6}\left\{f_{t}{ }^{2}-\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right. \\
& \left.\left[\frac{\left[\left(\rho_{1}+{ }^{4} \rho_{2}{ }^{2}-6 \rho_{1}{ }^{2} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\right\}\left(3 a^{2} h+a h^{2}(3+6 t)+\right. \\
& \left.h^{3}\left(3 t^{2}+3 t+1\right)\right) \\
& \frac{1}{24}\left[f_{t}{ }^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}{ }^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\left(4 a^{3} h+\right. \\
& \left.6 a^{2} h^{2}(1+2 t)+4 a h^{3}\left(3 t^{2}+3 t+1\right)+h^{4}\left(4 t^{3}+6 t^{2}+4 t+1\right)\right)
\end{aligned}
$$



$$
\begin{equation*}
+\frac{\left[e^{\rho_{1} h}\left(\cos \theta_{t} \cos \rho_{2} h-\sin \theta_{t} \sin \rho_{2} h\right)\right]-\cos \theta_{t} f_{t}^{4}}{R_{t}\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]} \tag{1.10}
\end{equation*}
$$

### 2.1 Definition of Terms

## Definition 1.1

The one step scheme (1.10) is said to be absolutely stable at a point $z$ in the complex plane provided the stability function or polynomial $\mu(z)$ fulfills the following conditions $|\mu(z)|<1$ and the region of absolute stability is defined as

$$
\begin{equation*}
R A S=\{z:|\mu(z)|<1\} \tag{1.11}
\end{equation*}
$$

## Definition 1.2

The numerical integration scheme (1.10) is said to A-stable provided the region of absolute stability specified in [2] includes the entire left half of the complex $z$-plane.

A-stability is very desirable property for any numerical integration algorithm, particularly if the IVP (1.17) is stiff or stiff oscillatory (i.e. an inherently stable differential equation system (1.17) in which the interval of integration is large).

### 2.2 Derivation of the Stability Function

To obtain the stability function for the new scheme (1.10), we proceed as follows

$$
\begin{align*}
& A_{1}=\left\{f_{t}-f_{t}^{1}-\left[f_{t}^{2}-\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}^{4}+\rho_{2}^{4}-6 \rho_{1}^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}^{3}-\rho_{1}^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}^{4}-10 \rho_{1}^{3} \rho_{2}^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right.\right. \\
& \left.\frac{\left[\left(\rho_{1}^{3}-3 \rho_{1}^{2} \rho_{2}^{2}\right) \operatorname{co} \theta_{t}+\left(6 \rho_{2}^{3}-3 \rho_{1}^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}^{4}-10 \rho_{1}^{3} \rho_{2}^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2} \\
& -\frac{1}{2}\left[f_{t}^{2}-\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}^{4}+\rho_{2}^{4}-6 \rho_{1}^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}^{3}-\rho_{1}^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}^{4}-10 \rho_{1}^{3} \rho_{2}^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right. \\
& \left.\left[\frac{\left[\left(\rho_{1}^{3}-3 \rho_{1} \rho_{2}^{2}\right) \operatorname{co} \theta_{t}+\left(6 \rho_{2}^{3}-3 \rho_{1}^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}^{4}-10 \rho_{1}^{3} \rho_{2}^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2}-f_{t}^{3}\right\} h  \tag{1.12}\\
& -\left\{\frac{1}{6}\left[\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{3}\right. \\
& \left.+\frac{\left[\rho_{1} \cos \theta_{t}-\rho_{2} \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}{ }^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right\} h
\end{align*}
$$

$$
\begin{align*}
& A_{2}=\frac{1}{2}\left\{f_{t}^{1}-\left[f_{t}{ }^{2}-\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)-\right.\right. \\
& \left.\frac{\left[\left(\rho_{1}^{3}-3 \rho_{1}{ }^{2} \rho_{2}{ }^{2}\right) \operatorname{co} \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}{ }^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+\text { th }) \\
& \left.-f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)^{2}- \\
& \left.\left[\frac{\left[\left(\rho_{1}^{2}-\rho_{2}^{2}\right) \cos \theta_{t}-2 \rho_{1} \rho_{2} \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\left(2 a h+(1+2 t) h^{2}\right)\right\} \tag{1.13}
\end{align*}
$$

$$
\begin{align*}
& A_{3}=\frac{1}{6}\left\{f_{t}^{2}-\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right](a+t h)\right. \\
& \left.-\left[\frac{\left[\left(\rho_{1}+{ }^{4} \rho_{2}{ }^{2}-6 \rho_{1}{ }^{2} \rho_{2}{ }^{2}\right) \operatorname{co~} \theta_{t}+\left(\rho_{2}{ }^{3}-3 \rho_{1}{ }^{2} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}{ }^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\right\}\left(3 a^{2} h+a h^{2}(3+6 t)\right.  \tag{1.14}\\
& \left.+h^{3}\left(3 t^{2}+3 t+1\right)\right)
\end{align*}
$$

$$
\begin{align*}
& A_{4}=\frac{1}{24}\left[f_{t}^{3}-\frac{\left[\left(\rho_{1}{ }^{4}+\rho_{2}{ }^{4}-6 \rho_{1}{ }^{2} \rho_{2}\right) \cos \theta_{t}+4\left(\rho_{1} \rho_{2}{ }^{3}-\rho_{1}{ }^{3} \rho_{2}\right) \sin \theta_{t}\right] f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1}^{3} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}^{2} \rho_{2}{ }^{3}-5 \rho_{1}^{4} \rho_{2}\right) \sin \theta_{t}\right]}\right]\left(4 a^{3} h+\right.  \tag{1.15}\\
& \left.6 a^{2} h^{2}(1+2 t)+4 a^{3}\left(3 t t^{2}+3 t+1\right)+h^{4}\left(4 t^{3}+6 t^{2}+4 t+1\right)\right)
\end{align*}
$$

$$
\begin{equation*}
A_{5}=\frac{\left[e^{\rho_{1} h}\left(\cos \theta_{t} \cos \rho_{2} h-\sin \theta_{t} \sin \rho_{2} h\right)\right]-\cos \theta_{t} f_{t}^{4}}{\left[\left(\rho_{1}^{5}+5 \rho_{1} \rho_{2}{ }^{4}-10 \rho_{1} \rho_{2}{ }^{2}\right) \cos \theta_{t}+\left(10 \rho_{1}{ }^{2} \rho_{2}{ }^{3}-5 \rho_{1}{ }^{4} \rho_{2}\right) \sin \theta_{t}\right]} \tag{1.16}
\end{equation*}
$$

We proceed to expand the above terms with the fact that

$$
\begin{aligned}
& f(x, y)=\lambda y \\
& f_{t}=\lambda y_{t} \\
& f_{t}^{1}=\lambda f_{t}=\lambda\left(\lambda y_{t}\right)=\lambda^{2} y_{t} \\
& f_{t}^{2}=\lambda f_{t}^{1}=\lambda\left(\lambda^{2} y_{t}\right)=\lambda^{3} y_{t} \\
& f_{t}^{3}=\lambda f_{t}^{2}=\lambda\left(\lambda^{3} y_{t}\right)=\lambda^{4} y_{t}
\end{aligned}
$$

Using the Maclaurin series of $e^{\rho h}, \cos \rho h$ and $\sin \rho h$ respectively, and then on simplification, equation (1.10) yields

$$
y_{t+1}=y_{t}+\lambda h_{t}+\frac{1}{2} h^{2} \lambda^{2} y_{t}+\frac{1}{6} h^{3} \lambda^{3} y_{t}+\frac{1}{24} h^{4} \lambda^{4} y_{t}
$$

$$
\begin{align*}
& y_{t+1}=\left(1+\lambda h+\frac{1}{2} h^{2} \lambda^{2}+\frac{1}{6} h^{3} \lambda^{3}+\frac{1}{24} h^{4} \lambda^{4}\right) y_{t}  \tag{1.17}\\
& \frac{y_{t+1}}{y_{t}}=\left(1+\lambda h+\frac{1}{2} h^{2} \lambda^{2}+\frac{1}{6} h^{3} \lambda^{3}+\frac{1}{24} h^{4} \lambda^{4}\right)
\end{align*}
$$

We define $z=\lambda h$, and then we have

$$
\begin{equation*}
\mu(z)=\frac{y_{t+1}}{y_{t}}=\left(1+z+\frac{1}{2} z^{2}+\frac{1}{6} z^{3}+\frac{1}{24} z^{4}\right) \tag{1.18}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\mu(z)=\left[1+z+\frac{1}{2} z^{2}+\frac{1}{6} z^{3}+\frac{1}{24} z^{4}\right] \tag{1.19}
\end{equation*}
$$

Equation (1.18) is the derived stability function for the scheme [11].
Comparing [11] with [2], [8] and [10]; [2] proposed a numerical integration whose method was based on the local representation of the theoretical solution $y(x)$ to the initial value problem of the form $y^{\prime}=f(x, y), y(a)=\eta$ in the interval $\left(x_{t}, x_{t+1}\right), t \geq 0$ by the polynomial interpolating function.

$$
F(x)=a_{0}+a_{1} x+b\left(\text { reale }^{\rho x+\mu}\right)
$$

Where $a_{0}, a_{1}$ and $b$ are real undetermined coefficients, $\rho$ and $\mu$ are complex parameters, he was able to derive the stability function as

$$
\begin{equation*}
\mu(z)=[1+z] \tag{1.20}
\end{equation*}
$$

[8] Improved on [2] a new numerical integration scheme also suited for the solution of the initial value problem $y^{\prime}=f(x, y), y(a)=\eta$ in the interval $\left(x_{t}, x_{t+1}\right), t \geq 0$ by the polynomial interpolating function.
$F(x)=a_{0}+a_{1} x+a_{2} x^{2}+b\left(\right.$ reale $\left.^{\rho x+\mu}\right)$
Where $a_{0}, a_{1}, a_{2}$ and $b$ are real undetermined coefficients, $\rho$ and $\mu$ are complex parameters, he was able to derive the stability function as
$\mu(z)=\left[1+z+\frac{1}{2} z^{2}\right]$
(1.21)
[10] is an improvement on [8] for the solution of the initial value problem of the form $y^{\prime}=f(x, y), y(a)=\eta$ in the interval $\left(x_{t}, x_{t+1}\right), t \geq 0$ by the polynomial interpolating function. $F(x)=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+b\left(\right.$ reale $\left.^{\rho x+\mu}\right)$

Where $a_{0}, a_{1}, a_{2}, a_{3}$ and $b$ are real undetermined coefficients, $\rho$ and $\mu$ are complex parameters, he was able to derive the stability function as
$\mu(z)=\left[1+z+\frac{1}{2} z^{2}+\frac{1}{6} z^{3}\right]$
(1.22)

### 3.0 CONCLUSION

Comparing the stability function of [2], [8], [10] and the newly derived one, it is obvious that the new stability polynomial is an improvement on the previous work of the three Scientists. With stability polynomial of higher degree by newly derived stability function gives an assurance that the scheme developed will be more stable and reliable.

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