APPLICATIONS AND ITS A SHORT HISTORY OF PROBABILITY THEORY

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It is remarkable that a science (Probability) which began with consideration of games of chance, should have become the most important object of human knowledge."

"Probability has reference partly to our ignorance, partly to our knowledge. The Theory of chances consist in reducing all events of the same kind to a certain number of cases equally possible, that is, such that we are *equally undecided* as to their existence; and determining the number of these cases which are favourable to the event sought. The ratio of that number to the number of all the possible cases is the measure of the probability"

P.S. Laplace

"The true logic of this world is to be found in theory of probability."

James Clark Maxwell

Abstract: This paper deals with a brief history of probability theory and its applications to Jacob Bernoulli's famous law of large numbers, theory of errors in observations, physics and the distribution of prime numbers. Included are major contributions of Jacob Bernoulli and Laplace. It is written to pay the tricentennial tribute to Jacob Bernoulli, since this year 2013 marks the tricentennial anniversary of the Bernoulli's law of large numbers since its publication in 1713. Special attention is given to the Bayes celebrated theorem and the famous controversy between the Bayesian and frequentism approaches to probability and statistics. This article is also written to pay a special tribute to Thomas Bayes since this year 2013 marks the 250th anniversary of the Bayes celebrated work in probability and statistics, since its posthumous publication in 1763. This is followed by the modern axiomatic theory of probability first discovered by A.N. Kolmogorov in 1933. The last section is devoted to applications of probability theory to the distribution of prime numbers.

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Ancient Origins of the Concept of Probability:

In ancient times, Plato (428-348 BC) and his famous student, Aristotle (384-322 BC) used to discuss the word *chance* philosophically. In 324 BC, a Greek person, Antimenes (530-510 BC) first developed the system of insurance which guaranteed a sum of money against wins or losses of certain events. In view of many uncertainties of everyday life such as health, weather, birth, death and game that led to the concept of chance or random variables as output of an experiment (for example, the length of an object, the height of people, the temperature in a city in a given day). Almost all measurements in mathematics or science have the fundamental property that the results vary in different trials. In other words, results are, in general, random in nature. Thus, the quantity we want to measure is called a *random variable*.

Historically, the word probability was associated with the Latin word '*probo*' and the English words, *probe* and *probable*. In other languages, this word used in a mathematical sense had a meaning more or less like *plausibility*. In ancient times, the concept of probability arose in problems of gambling dealing with winning or losing of a game. It began with famous physician, mathematician and gambler, Gerolamo Cardano (1501-1576) who became Professor and Chair of mathematics at the University of Bologna in Italy. During the fifteenth century, the pragmatic approach to problems of games of chance with dice began in Italy. During that time, references to games of chance were more numerous, but no suggestions were made how to calculate probabilities of events. Cardano wrote a short manual, *Liber de Ludo Aleae* (*Games of Chance*) which contained the first mathematical treatment of probability dealing with problems of mathematical expectation (or mean), addition of probability, frequency tables for throwing of a dice, n successes in n independent trials, and the law of large numbers. However, his work attracted a little attention and did not provide any real development of probability theory. Cardano's manual was published in 1633 about a century later. In this published manual, Cardano introduced the idea of probability p between 0 and 1 to an event whose outcome is random, and then applied this idea to games of chance. He also developed the law of large numbers which states that when the probability of an event is p, then after a large number of trials n, the number of times it will occur is close to np.

Development of Classical Probability During the Sixteenth and Seventeenth Centuries:

During the sixteenth and seventeenth centuries, a great deal of attention was given to games of chance, such as tossing coins, throwing dice or playing cards, in particular, and to problems of gambling, in general. An Italian nobleman suggested a problem of throwing dice to Galileo Galilei (1564-1642), a great Italian astronomer and physicist, a solution of which was the first recorded result in the history of mathematical probability theory. A decade after Galileo's death, a French nobleman and gambling expert, Chevalier de Mére (1610-1685) proposed some mathematical questions on games of chance to Baise Pascal (1623-1662), French mathematician, who communicated these to another French mathematician, Pierre de Fermat (1601- 1665). From 1654, both Pascal and Fermat began a lively correspondence about questions and problems dealing with games of chance, arrangements of objects, and chance of winning a fair game. Their famous correspondence introduced the concept of probability, mean (or expected) value and conditional probability and hence, can be regarded as the mark of the birth of classical probability theory. According to Pascal, when a coin is tossed twice, there are four possible outcomes: *HH*, *HT*, *TH* and *TT* where *H* stands for heads and *T* for tails, a solution of which was closely related to the binomial coefficients of

$$(H+T)^2 = H^2 + HT + TH + T^2 = H^2 + 2HT + T^2. \quad (2.1)$$

Similarly, when an ideal coin is tossed three times, there are eight possible outcomes: *HHH*, *HHT*, *HTH*, *THH*, *HTT*, *THT*, *TTH*, *TTT*. A solution of which was closely linked with the binomial coefficients (collecting all terms of *H* and *T* together) of

$$(H+T)^3 = H^3 + 3H^2T + 3HT^2 + T^3. \quad (2.2)$$

The binomial coefficients in (2.2) led to the famous *Pascal triangle* as shown in Figure 1. In connection with the Pascal triangle as shown in Figure 1, Pascal observed the famous formula for the binomial coefficients n=0 1

n=1 1 1

n=2 1 2 1

n=3 1 3 3 1

n=4 1 4 6 4 1

n=5 1 5 10 10 5 1

n=6 ∘ ∘ ∘ ∘ ∘ ∘ ∘

Figure 1: Pascal triangle: coefficients of bionomial formulas for $n = 0, 1, 2, 3, 4, 5, \cdots$.

${}^{n}C_r = \binom{n}{r}$ involved in the coefficient of $a^r b^{n-r}$ in the binomial expansion of $(a + b)^n$ for any integer $n \geq 0$ and $0 \leq r \leq n$. He also linked binomial coefficients with the combinatorial coefficients that arose in probability, and used these coefficients to solve the problem of points in the case of one player requires r points and the other n points to win a game. At the same time, Pascal also wrote his *'Traite' du triangle arithmétique* (*Treatise on the Arithmetical Triangle*) in which he found a triangular arrangement of the binomial coefficients and proved many of their new properties. In this work, he gave what seemed to be the first satisfactory statement of the principle and proof by mathematical induction (see Debnath [1]). However, the Pascal triangles for $n = 1, 2, , 8$ was found earlier in the work of Chinese mathematician, Chu Shih-Chieh in 1303.

With the ancient rudiments provided by Cardano a century before, the Pascal and Fermat's work laid the classical mathematical foundation of the probability theory in the middle of the seventeenth century. Stim- ulated by the significant advances of Pascal and Fermat, Christian Huygens (1629-1695), an extraordinary mathematical physicist and astronomer of Holland, became actively involved in the study of probability, and published the first treatise on probability theory "*De Rationiis in Ludo Aleae* (*On Reasoning in Games of Dice*)" in 1657. This major treatise contained the logical treatment of probability theory, and many impor- tant results including problems of throwing of dice, chance of winning games, problems of games involving three or four players in addition to some works of Pascal and Fermat. In his treatise, Huygens introduced the definition of probability of an event as a quotient of favorable cases by all possible cases. He also rein- troduced the concept of mathematical expectation (or mean) and elaborated Cardano's idea of expected value (or mean) of random variables. In those days and even in current days, there was a disagreement among different disciplines on the meaning and significance of the terms chance, probability, likelihood and uncertainty. However, Huygen's treatise provided the greatest impetus to the subsequent development of the probability theory, and remained the best book on probability theory until the publication of Jacob Bernoulli's (1654-1705) first significant work "*Ars Conjectandi* (*Arts of Prediction*)" on probability in 1713 eight years after his death. So, Huygen may perhaps be regarded as the father of the theory of probability. In addition, to many major ideas and results included in his monumental treatise, Bernoulli formulated the fundamental principle of probability theory, known as *Bernoulli's theorem* or *the law of large numbers*, a name given by a celebrated French mathematician, Siméon Poisson (1781-1840).

The Bernoulli's law of large numbers is so fundamental that the subsequent development of probability theory is based on the law of large numbers. In his letter to Gottfried Wilhelm Leibniz (1646-1716) on October 3, 1703, Jacob Bernoulli described the major contents of his *Ars Conjectandi* and stated that the most of it has been completed except applications to civil, moral, social and economic problems, and to uncertainties of life beyond the analysis of games of chance. From the time of Bernoulli and even today, the famous phrase such as *proof beyond reasonable doubt* in law involve probability.

However, he discussed the applications of probability theory in great details. Also included in his book were some problems as well as solutions of Huygens with his own ideas and solutions, problems of throwing dice and games of chance. In particular, he added the problem of repeated experiments in which a particular outcome either a success or